Persuasive Advertising and Welfare

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CRETE Conference 2026

backdrop

Most ads in “tight” oligopoly

Most ad theories developed for monopoly, some extensions (combative, signaling, complementary goods,…)

Search pulls up little theory literature on persuasive ad levels and oligopoly (quality provision)

Grossman-Shapiro (1984) informative ads, circle model

Herewith a suite of asymmetric oligopoly models & techniques

keywords

- Logit, IIA, rum
- Aggregate games
- Combative/persuasive ads
- Zero-sum ads
- Ads exacerbating dominance, concentration
- Playing to strength

- (when/how) does persuasive advertising leverage dominance?

Baseline model: Binomial Logit (covered market)

- Two firms *i = 1, 2* compete for consumers
  Intrinsic valuation V_i ; heterogeneity $\mu > 0$
- Advertising effort: $x_i \geq 0$
- Persuasion technology: $a(x_i)$, increasing and concave

Choice probabilities (logit):

$$\mathbb{P}_i = \frac{\exp(Vi + a(xi))/\mu}{\exp(Vj + a(xj)/\mu + \exp(Vi + a(xi))/\mu} = \frac{\boldsymbol{\alpha_i}}{\boldsymbol{A}}$$

One micro-foundation for the choice model is in discrete choice theory, writing

$u_i = V_i + a(x_i) + \mu\varepsilon_i$, $i=1,2$

with the ε_i i.i.d. standardized Type 1 Extreme Value (Gumbel) distributed idiosyncratic match terms (see Anderson, de Palma, and Thisse, 1992, Ch.2 for more background). Or indeed, we can write

$u_1 = \theta + a(x_1) - a(x_2) + \mu\varepsilon$

$u_2 = 0$

with ε logistically distributed.

Zero-Sum Advertising

- Profit per consumer: $\pi_i = \mathbb{P}_i - \lambda x_i$
- Interior foc:

$\mathbb{P}_i(1 - \mathbb{P}_i)\, a'(x_i) = \lambda\mu$

- Covered market duopoly (also dcm's) implies

$\mathbb{P}_1(1 - \mathbb{P}_1) = \mathbb{P}_2(1 - \mathbb{P}_2)$

- Drives zero-sum ad neutrality/cancellation

Same ad levels and firms stay where they were without ads

Old saw in ad discussion (e.g. cigarettes)

Property: "The common level of advertising is a strictly decreasing function of $|V_2 - V_1|$"

- from $\mathbb{P}_i(1 - \mathbb{P}_i)$ hillock

More ads the tighter the duopoly

Welfare for covered market

- Extreme views (Dixit – Norman 1977 Bell)

- Reshuffle, no explicit gains: too much ads

If advertising affects choices but not fundamental utilities, advertising has no effect on consumer welfare but only increases firms' costs and hence decreases social surplus.

Welfare (II)

If ads enhance consumer appreciation of what they consume (e.g. Becker-Murphy-Stigler complementary good) then too few ads:

If advertising affects choices and creates value to consumers, consumers are better off with advertising but firms are worse off. Moreover; advertising is both insufficient and optimally asymmetric: the better firm should advertise more than the worse one.

i) if ads create value as per the function a(x) then consumers are better off the more ads there are,

but firms are worse off by the amount of the ad-spend.

ii) equilibrium entails equal advertising with foc

$\mathbb{P}_1(x_1,x_2)(1- \mathbb{P}_1(x_1,x_2))a'(x_1)=\lambda\mu$.

But the welfare derivative is

$\mathbb{P}_i\, a'(x_i)/\mu - \lambda$ which is positive

Moreover, because $\mathbb{P}_1 > \mathbb{P}_2$, the firm with the initial advantage should advertise more: the incremental ad is only enjoyed by those buying

n persuaders, outside option

Same foc holds:

When interior,

$\mathbb{P}_i(1 - \mathbb{P}_i)\, a'(x_i) = \lambda\mu$

But now $(1 - \mathbb{P}_i) > \mathbb{P}_j$ …

Indeed, with an active outside option (&/or n>2)

$\mathbb{P}_1 > \mathbb{P}_2 \Leftrightarrow \mathbb{P}_1(1- \mathbb{P}_1) > \mathbb{P}_2(1- \mathbb{P}_2)$

with an active outside option (&/or n>2):

$$\mathbb{P}_1 > \mathbb{P}_2 \Leftrightarrow \mathbb{P}_1(1- \mathbb{P}_1) > \mathbb{P}_2(1- \mathbb{P}_2)$$

$\mathbb{P}_1(1- \mathbb{P}_1)$ is a quadratic form,
increasing for $\mathbb{P}_1 < 1/2$ and symmetric around 1/2.

So for $\mathbb{P}_1 < 1/2$ the relation is clearly true.
Indeed it is true in general: because
$\mathbb{P}_1(1- \mathbb{P}_1) > \mathbb{P}_2(1- \mathbb{P}_2) \Leftrightarrow \mathbb{P}_1 - \mathbb{P}_2 > \mathbb{P}_1^2 - \mathbb{P}_2^2$
which is true for $\mathbb{P}_1 > \mathbb{P}_2$ if and only if $1 > \mathbb{P}_1 + \mathbb{P}_2$.

If instead $\mathbb{P}_0=0$, as before, then we have $\mathbb{P}_1(1 - \mathbb{P}_1)= \mathbb{P}_2(1- \mathbb{P}_2)$ which was the property we used before.

We will use this Lemma for characterization results

More win from more ads

- With more than two options, we have:

$V_1+a(x_1) > V_2+a(x_2) \Leftrightarrow \mathbb{P}_1 > \mathbb{P}_2$

$\Leftrightarrow a'(x_1) < a'(x_2) \Leftrightarrow x_1 > x_2 \Leftrightarrow a(x_1) > a(x_2)$

- By last Lemma

$\mathbb{P}_1 > \mathbb{P}_2 \Leftrightarrow \mathbb{P}_1(1- \mathbb{P}_1) > \mathbb{P}_2(1- \mathbb{P}_2) \Leftrightarrow a'(x_1) < a'(x_2)$ where the last one comes from the focs for the problem:

$\mathbb{P}_i(1 - \mathbb{P}_i)\, a'(x_i) = \lambda\mu$

By concavity and monotonicity of a(.),

$a'(x_1) < a'(x_2) \Leftrightarrow x_1 > x_2 \Leftrightarrow a(x_1) > a(x_2)$.

Ad rankings reflect quality rankings (hence leverage initial advantages)

Last slide tells us that higher sales & post-ad quality is more ads

But how do we know if it is also pre-ad higher quality?

We can trace equilibrium actions as a cts function of λ

For λ high enough there are no ads

Highest quality is “first” entrant, firms “enter” in order

There cannot be a flip in relative ad intensities:

At any possible crossing point we would have 2 firms with different qualities but same size and same ads!

Aggregate game approach will do this nicer

(when/how) does persuasive advertising increase dominance?

- the log-odds ratio.
- *n* firms, outside good (not duopoly covered)

$$\ln(\mathbb{P}_i / \mathbb{P}_j) = V_i + a(x_i) - (V_j + a(x_j))$$

so advertising raises market concentration

By leveraging prior advantages

holds for any pair $x_i > x_j$ as $V_i > V_j$

Existence (presage Aggregative Game)

Rewrite from $\mathbb{P}_i = \dfrac{\exp(Vi + a(xi))/\mu}{\Sigma_j \exp(Vj + a(xj)/\mu + \exp V_0/\mu} = \dfrac{\boldsymbol{\alpha_i}}{A}$

so profit as $\dfrac{\boldsymbol{\alpha_i}}{A} - \lambda a^{-1}(\mu \ln \boldsymbol{\alpha_i} - V_i)$

first term is concave : so seek sufficient condition for the cost term. Second derivative is $x''(.)(\mu/\boldsymbol{\alpha_i})^2 - x'(.)\mu/\boldsymbol{\alpha_i}^2$

Suffices that $((x''(.))/(x'(.))) > 1/\mu$

example $x=\exp(ka) - 1$ so $x'=k\exp(ka)$ & $x''=k^2\exp(ka)$

OK for $k > 1/\mu$ (inverting, $a(x)=(1/k)\ln(1+x)$)

So μ should be large enough

For MSE:

- If μ =0; no PSE. Consider 1st duopoly
- weaker must ad to survive, rival can "undercut":

in turn undercut, like Bertrand-Edgeworth cycle;

raising ads along the pseudo-dynamic

but then stop when weak makes nothing

- Support of MSE for weaker is bounds of the "cycle"

- $V_1 = V_2 + a(x_2^L)$ and $a(x_2^H) = 1$ (market worth)
- And $a(x_2^L)$ is endogenous entry cost

MSE properties, *n* asymmetric firms

- "It takes 2 to tango" (Model of Sales)

Baye, Kovenock, de Vries GEB92
- only 2 players active in eqm
top guy gets rents,
second nothing but must enter to keep first honest;
all others stay out (do nothing)

- "Personalized Pricing" Anderson Baik Larson GEB15
- Varian MoS, Myatt & Ronayne
- Bertrand-Edgeworth:
- Contests, etc.

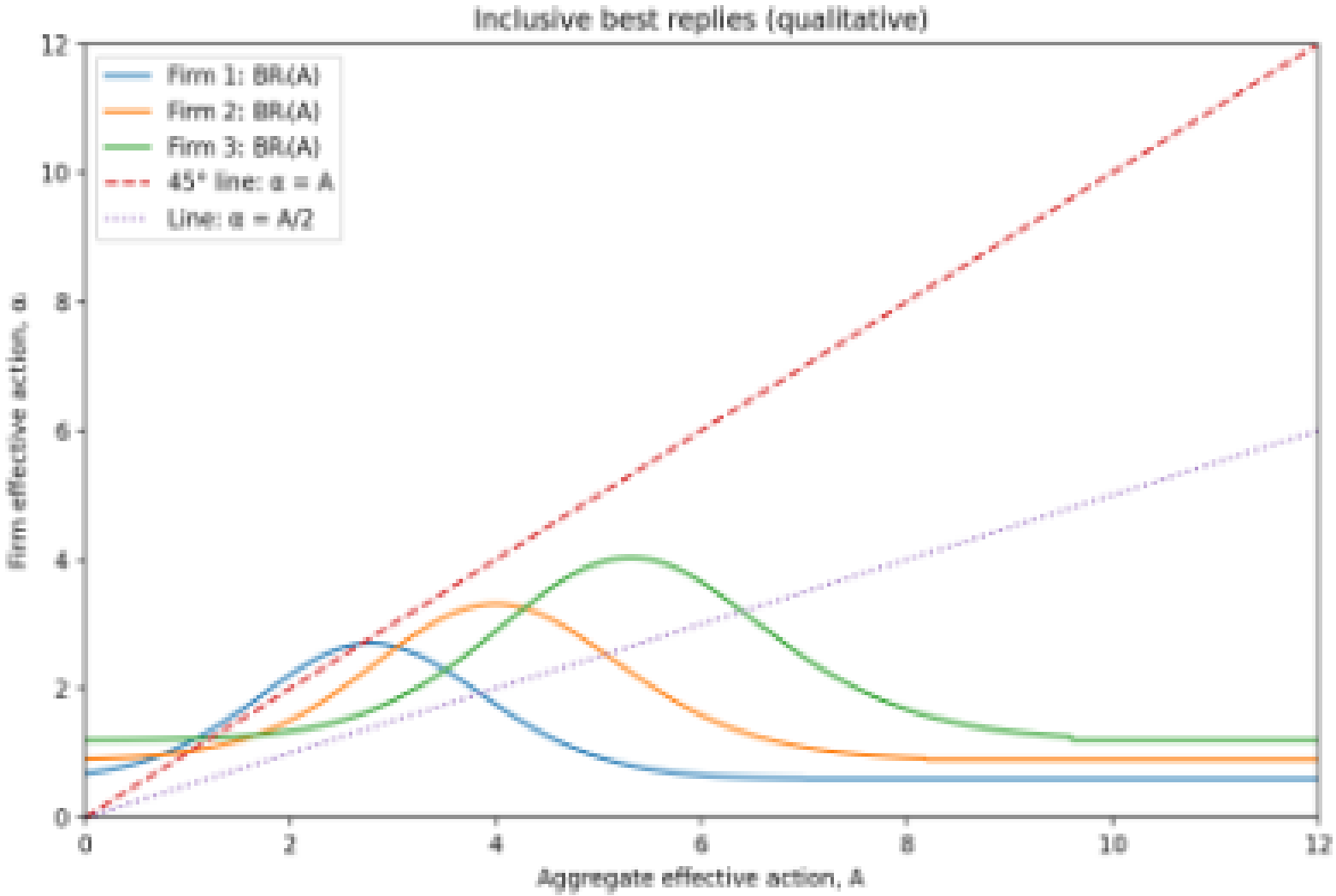
Aggregate Games Oligopoly

- We can write profit as a function of own action and the sum of others' actions: AG

- Clear and immediate results for asymmetric firms
- Selten (1971)
- Corchon (1993) Cornes and Hartley
- Anderson, Erkal, Picinnin (2013, 2020)
- Nocke & Schutz

Showcase here

Inclusive best replies



IBR (fitting-in functions) in Agg Game: equilibrium actions and properties

Use ibr-foc (marginal profit), e.g. with prior cost function,

$[\frac{\mathbf{1}}{\boldsymbol{A}} - \frac{\boldsymbol{\alpha_i}}{\boldsymbol{A^2}}] - \lambda k\mu \boldsymbol{\alpha_i}^{k\mu-1}\exp(-kV_i)$, which is marginal profit.

- marginal profit increases in **A** (an increasing ibr / strategic complements) if d[.]/d**A** > 0 as $\boldsymbol{\alpha}_i > \mathbf{A}/2$
- this means $\boldsymbol{\alpha}_i$ rising with **A** when **A** is low and relation $\boldsymbol{\alpha}_i\mathbf{(A)}$ is maximal at $\boldsymbol{\alpha}_i = \mathbf{A}/2$

- Ranking of actions by quality tells higher quality does more ads and gets more sales

Hence ads leverage advantages

- Also read off comparative statics
- No ads if **A** high enough (per quality)

Endogenous prices

- Profit: $(p_i\text{-}c_i)\mathbb{P}_i - \lambda x_i$

- Logit price foc: $(p_i\text{-}c_i) = \mu/(1 - \mathbb{P}_i)$
- And ads : $(p_i\text{-}c_i)\ \mathbb{P}_i(1 - \mathbb{P}_i)\ a'(x_i) = \lambda\mu$

- Hence $\mathbb{P}_i\ a'(x_i) = \lambda$ (hence firm rankings)

Or indeed more directly and generally:
A small ad increase can be "neutralized" by offsetting price rise at rate a'(x)

Thus (by earlier methods):
At a simultaneous price-and-persuasive advertising equilibrium, equilibrium mark-ups, sales, profits, and advertising levels follow the ranking of initial firm qualities

(constrained) optimal ad levels! When ad spending delivers full benefits

Related: A sheening theory of advertising

SA with Federico Ciliberto (UVa) and Jura Liaukonyte (Cornell)

Ads “polishing” characteristics in goods

Micro-founding persuasion at characteristics level

Spillovers to products with similar characteristics

Lancaster model embodied in discrete choice framework, like BLP

- Light beer ad makes thirst for lo-cal brew

Sheening demands

DCM Conditional (indirect) utility $u_i = q_i - p_i + \varepsilon_i$

E.g. Logit : $\mathbb{P}_i = \exp(q_i - p_i)/(\Sigma_j \exp(q_j - p_j) + \exp V_0)$

- $q_i = \Sigma_k \beta^k X_i^k \phi_k(a^k)$ (hedonic, Lancaster, random coeffics)

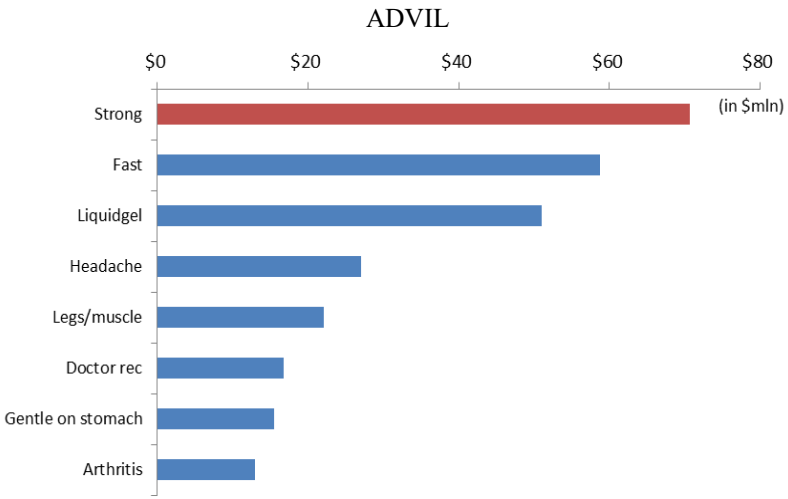
- β^k as base characteristic valuation
- X_i^k as characteristic endowment in brand i
- $\phi_k(a^k)$ as strictly concave sheening function ($\phi(0) = 1$)
- a^k as $ promoting the characteristic

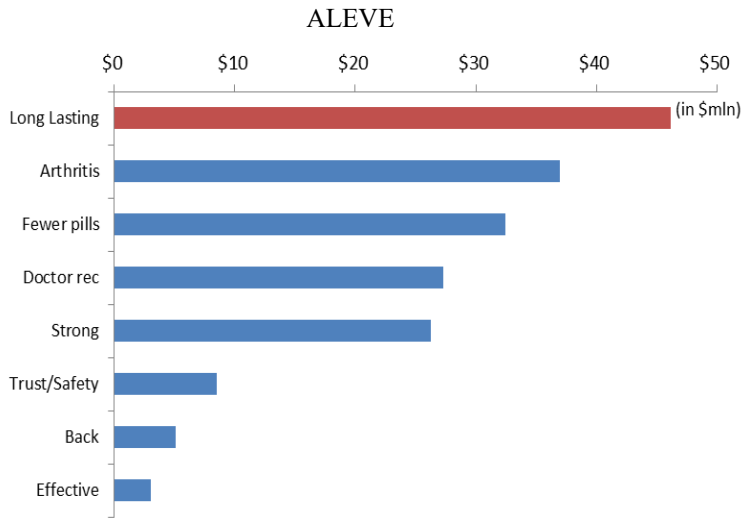
Brand profit $\pi_i = (p_i - c_i)\mathbb{P}_i - \Sigma_k a_i^k$

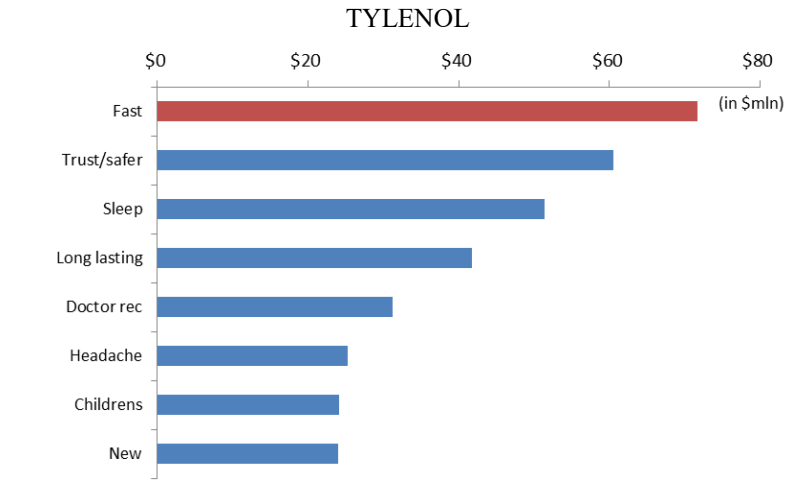
- Public Goods contribution game (e.g. MWG)
- One advertiser per characteristic

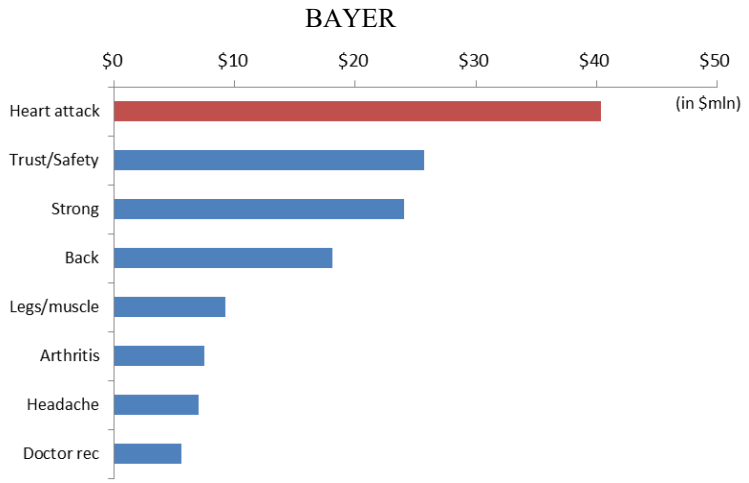
(simple diversity index statistic $W_i^k = (X_i^k - \Sigma_{j \neq i} d_{ji} X_j^k)$ with d_{ji} the Diversion Ratio from good i to j tells who does how much of what)

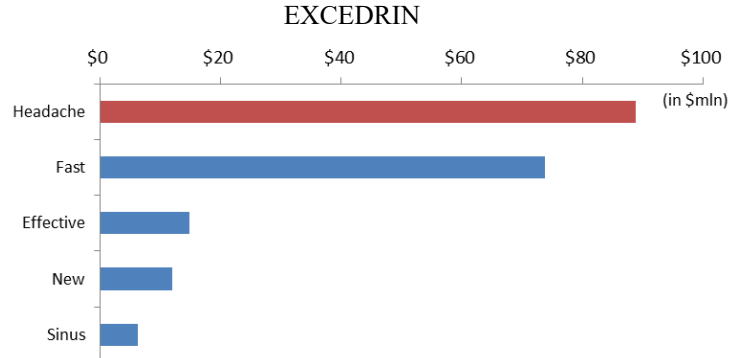
- Gives free-rider patterns, e.g. “raise all boats”

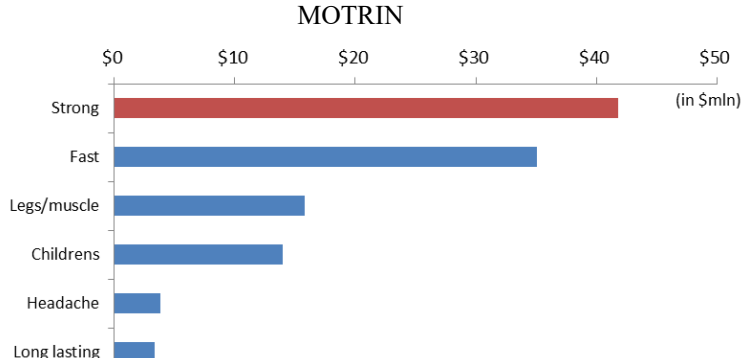

ADVIL
$0
$20
$40
$60
$80
(in $mln)
Strong
Fast
Liquidgel
Headache
Legs/muscle
Doctor rec
Gentle on stomach
Arthritis


ALEVE
$0
$10
$20
$30
$40
$50
(in $mln)
Long Lasting
Arthritis
Fewer pills
Doctor rec
Strong
Trust/Safety
Back
Effective


TYLENOL
$0
$20
$40
$60
$80
(in $mln)
Fast
Trust/safer
Sleep
Long lasting
Doctor rec
Headache
Childrens
New


BAYER
$0
$10
$20
$30
$40
$50
(in $mln)
Heart attack
Trust/Safety
Strong
Back
Legs/muscle
Arthritis
Headache
Doctor rec


EXCEDRIN
$0
$20
$40
$60
$80
$100
(in $mln)
Headache
Fast
Effective
New
Sinus


MOTRIN
$0
$10
$20
$30
$40
$50
(in $mln)
Strong
Fast
Legs/muscle
Childrens
Headache
Long lasting

- “ […] the fundamental aims of advertising are: the creation of wants, the shaping of tastes, the determination of values” (Stocking, 1931, AER).
- “demand for articles is being awakened, intensified, directed and made articulate.” (same source)

Conclusions & take-aways

- Ad neutral in duopoly covered market

(more ads if consumers more evenly split)

- Otherwise, dominance selectively amplified through ads
- Access to ads increases industry concentration

future directions

- Ads in tight oligopolies: hump-shape in n?
- More important in duopoly than monopoly
- Ads lose power and strength with more firms in market
- Dominant firm results?
- Ad cost and concentration: as function of λ
- Size distribution of firms and ads (AdP: EJ20, JET24)
- Two-stage game: persuasion then prices

**Targeted Persuasive Advertising

Simon P. Anderson & André de Palma TLS Jan 8

260108 eqs reposing:

$u_i = V_i + a(x_i) + \mu\varepsilon_i$, i=1,2

$D_i = \exp(q_i - \alpha p_i)/(\Sigma_j \exp(q_j - \alpha p_j) + \exp V_0)$

$$\mathbb{P}_i = \frac{\exp(Vi + a(xi))/\mu}{\Sigma_j \exp(V_j + a(xj)/\mu + \exp V_0/\mu} = \frac{\alpha_i}{A}$$

Binomial Logit (covered market):

$$\mathbb{P}_i = \frac{\exp(Vi + a(xi))/\mu}{\exp(Vj + a(xj)/\mu + \exp(Vi + a(xi))/\mu} = \frac{\alpha_i}{A}$$

Order of events, roadmap

- 1st shot is focs from older version
- Hence zero-sum property
- Nice panama hat comparative statics; fight more in middle!
- Robustness (e.g. rum, n –firm covered) ok
- First take existence: nice version of Aggregate-Game inspired technique (logit numerator – “action”)
- Existence iff condition, give e.g. exponential. Clean!
- Need enough mu for equilib
- Notice nice depiction of reaction functions, in numerator space (can we draw a picture?)
- For low rival action, strategic complements, but strategic substitutes for high action; because dividing line is 45-degree line there’s a unique equilibrium! And high advertises more
- Eqm Properties, n-firm covered/uncovered: ads exacerbate prior advantages and so raise concentration
- Proof: nice use of focs to show that high sales are high ad levels; then uses equilibrium mapping as function of lambda (pic might be useful)
- Welfare (maybe we only have duopoly covered market, but easily extends to n-firms covered)
- Aside: if existence condition not satisfied?: MSE
- E.g. mu=0: Quick properties of the equilib (“2 to tango”; list of similar problems with this result?)
- Are these general here, I don’t know what for mu>0