Market Segmentation in a Hotelling Model: The role of product substitutability

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A. INTRODUCTION

- More or less differentiated products?
  - Product substitutability usually limits a firms' ability to raise price-cost margins
  - However, firms may in some cases intentionally reduce product differentiation in order to:
    - **lower manufacturing costs** (economies of scale),
    - **enable price collusion (increase detectability of deviations)**,
    - **achieve industry-wide standard adoption**,
    - **replicate the market-leading product** to avoid losing mainstream customers (converge to the center of the Hotelling line)
- These explanations are based on differentiation over a **single product attribute**
- Richards, Allender, and Hamilton (AJAgrEcon 2013), found that in the yogurt market prices can increase when manufacturers choose more similar product attributes.
  - None of the explanations above can explain their finding

- In this work we show that, when product substitutability refers to multiple characteristics, reducing differentiation on one dimension may increase differentiation on the other
- This may help firms maintain their niche markets in the absence of information about specific consumer characteristics, like their location.

B. LITERATURE

- HOTELLING (1929)
  - Each consumer demands only one unit from a single firm.
  - The model implies no consumer preference for variety or loyalty.

- A. SMITHIES (*JPE,* vol 49, no 3, 1941), contests the unit demand assumption of Hotelling, by introducing a downward sloping demand at every point. Products are perfect substitutes if no for their location

- KIM & SERFES (2006) presents a model where consumers along a Hotelling's line may purchase one unit from one or both firms.
  - Substitute products, but unit demand.
    - Loyal consumers buy only 1 unit and get utility $V = \alpha$

- Diversity lovers buy 2 units one from each firm and get utility $V = \alpha + \gamma$.
  - $0 < \gamma < \alpha$ is the utility from consuming the 2nd product
  - The closer substitutes the two products are, the smaller is the value of γ (consuming two substitute products provides less utility than consuming two different products)
- While in this work the differentiation parameter γ may affect the presence or not of niche markets, equilibrium prices are always increasing with product differentiation

- LU & MATSUSHIMA (2024) uses a similar model to examine personalized prices (3rd degree price discrimination across locations).
- In this work we allow for both, substitutability (other than location) and continuous demand.

C. THE MODEL

- Assume a typical Hotelling's line between 0 and 1, with two rival products located at the extremes of the interval.
    - Let firm 1 be at the left and firm 2 at the right end of the interval.
- Each point on the line is inhabited by consumers consuming both goods at variable quantities.
    - For convenience, instead of defining consumers' addresses as distances from point 0, we define them as distances from the center of the interval (1/2).

- Consumers located at distance x_i from the center may be represented by a "representative consumer" with utility:

$$U_i = a\,(q_{1i} + q_{2i}) + \tau\left(\frac{1}{2} - x_i\right)(q_{1i} - q_{2i}) - \frac{b}{2}\left(q_{1i}^2 + q_{2i}^2 + 2\theta q_{1i}q_{2i}\right) + M$$

  - The above a typical Bowley U function to which a "transport cost disutility" has been added (blue term)

- The corresponding inverse demand functions for the two products are:

$$p_{1i} = a + \tau\left(\frac{1}{2} - x_i\right) - bq_{1i} - \theta bq_{2i}$$

$$p_{2i} = a - \tau\left(\frac{1}{2} - x_i\right) - bq_{2i} - \theta bq_{1i}$$

- Hotelling's line is considered as "horizontal differentiation" because there is no consumer unanimity upon their preferred product.

- However, it divides the market in two subsets with consumers exhibiting brand name loyalty (vertical differentiation) within each firm's neighborhood.
    - *E.g.,* for a consumer with $x_i > 1/2$, an amount $\tau\left(x_i - \frac{1}{2}\right) > 0$ is added to his willingness-to-pay for product 2 and the same amount is subtracted from his willingness-to-pay from product 1.
- Inverting the previous marginal w-t-p functions, we obtain the direct demand functions:

$$D_{1i}(p_1, p_2, x_i; \tau, \theta) = \frac{1}{1+\theta} + \frac{\tau}{1-\theta}\left(\frac{1}{2} - x_i\right) - \frac{1}{1-\theta^2}p_1 + \frac{\theta}{1-\theta^2}p_2$$

$$D_{2i}(p_1, p_2, x_i; \tau, \theta) = \frac{1}{1+\theta} - \frac{\tau}{1-\theta}\left(\frac{1}{2} - x_i\right) - \frac{1}{1-\theta^2}p_2 + \frac{\theta}{1-\theta^2}p_1$$

    - The term in red is the marginal impact of distance in the demand function
        - ***Effective marginal transport cost*** **(EMT)**
    - EMT is increasing in θ: an increase in product homogeneity reduces consumer's willingness to purchase the product of the more distant seller.

D. EQUILIBRIUM and DISCUSSION

- We need first to find the "critical consumer" for each firm, *i.e.,* the consumer who stops purchasing at all from the given firm. The condition for this is:

$$\hat{x}_1(p_1, p_2, x_i; \tau, \theta) = \frac{1}{2} + \frac{1 - \theta - p_1 + \theta p_2}{(1+\theta)\tau}$$

$$\hat{x}_2(p_1, p_2, x_i; \tau, \theta) = \frac{1}{2} - \frac{1 - \theta - p_2 + \theta p_1}{(1+\theta)\tau}$$

- Firm 1 sells in the interval $[0, \hat{x}_1]$ and firm 2 in the interval $[\hat{x}_2, 1]$.
- Note that since the critical consumers may be at different positions with $\hat{x}_1 > \hat{x}_2$ each firm sells at two segments:
  - a common market $(\hat{x}_2, \hat{x}_1)$
  - a niche segment which is $[0, \hat{x}_2]$ for firm 1 and $[\hat{x}_2, 1]$ for firm 2.

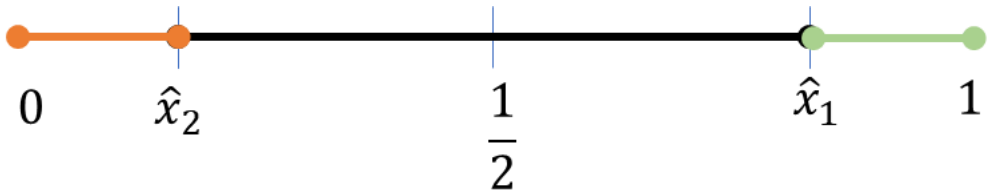


Figure 1: ***Potential market configuration***

- The demand functions in the niche markets (colored intervals) are:

$$D_{1i}^{M}\,(p_1, x_i;\ \tau) = 1 + \tau\left(\frac{1}{2} - x_i\right) - p_1, \qquad D_{2i}^{M}(p_2, x_i;\ \tau) = 1 - \tau\left(\frac{1}{2} - x_i\right) - p_2$$

- Given that no firm can identify the exact consumers' location, the profit functions are:

$$\Pi_1 = p_1\left(\int_0^{\hat{x}_2(\mathbf{p};\tau,\theta)} D_{1i}^{M}\,(p_1, x_i;\tau) f dx_i + \int_{\hat{x}_2(\mathbf{p};\tau,\theta)}^{\hat{x}_1(\mathbf{p};\tau,\theta)} D_{1i}\,(\mathbf{p}, x_i;\tau,\theta) f dx_i\right)$$

$$\Pi_2 = p_2\left(\int_{\hat{x}_2(\mathbf{p};\tau,\theta)}^{\hat{x}_1(\mathbf{p};\tau,\theta)} D_{2i}\,(\mathbf{p}, x_i;\tau,\theta) f dx_i + \int_{\hat{x}_1(\mathbf{p};\tau,\theta)}^{1} D_{2i}^{M}\,(p_2, x_i;\tau) f dx_i\right)$$

where $\mathbf{p} = (p_1, p_2)$

- Due to symmetry, after maximizing the above and solving the system of reaction functions, we obtain $p_1^* = p_2^* \equiv p^*$ where

$$p^* = \frac{(1+\theta)^2\tau + 2(\theta^2 - \theta + 1) - \Delta(\tau, \theta)}{3\theta^2 - 2\theta + 3}$$

- **Lemma 1:**
  - For low values of τ, $\tau \in (0,1)$, there exists a critical value of θ, call it $\tilde{\theta}$, such that $\forall\theta \in [\tilde{\theta}, 1)$ the width of the common market segment is less than 1.
    - Niche markets appear only when the two products are close substitutes
  - For intermediate values of $\tau \in (1,4)$, the niche markets exist independently of θ.
  - For $\tau > 4$, the common market disappears and the two firms become independent monopolies.

➢ We rule out the case of independent monopolies ($\tau > 4$). Obviously, the previously computed value of p^* holds only in presence of niche markets, so we assume hereafter parameter values such that the market configuration contains both, a common segment and niche markets, like in the figure presented earlier:

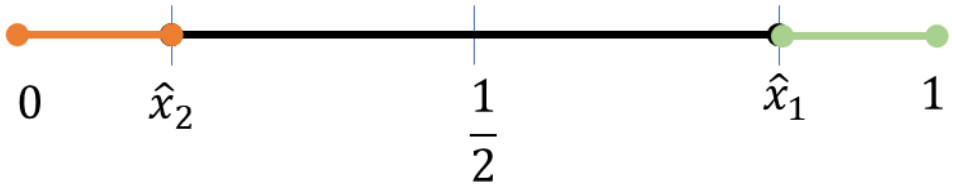
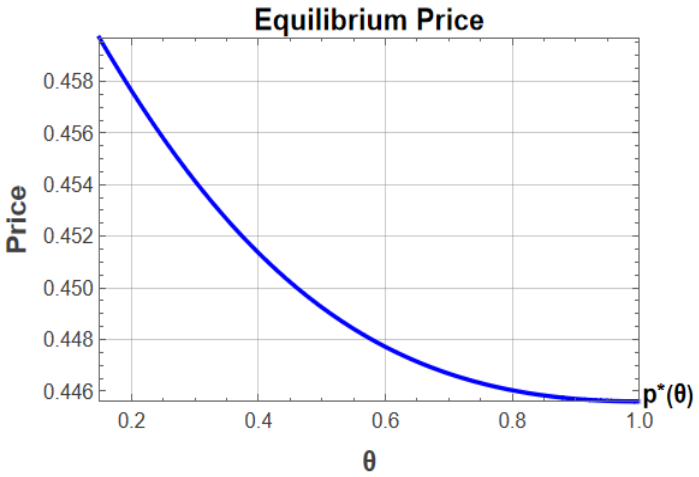
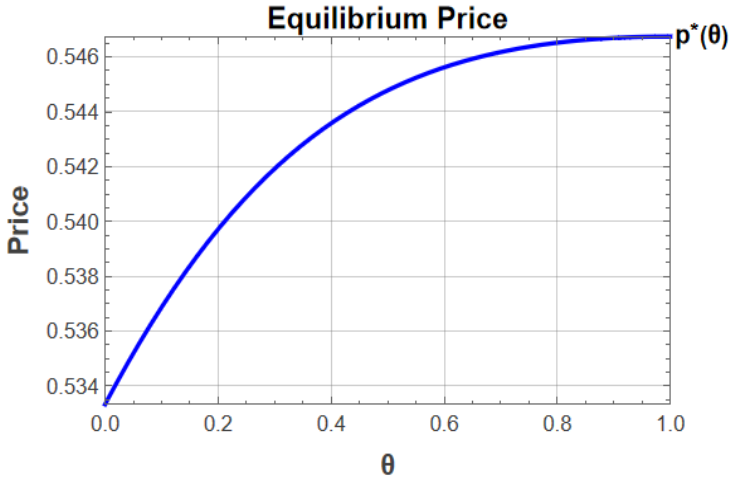


Figure 1 (repeat)**: *Potential market configuration***

- **Proposition 1:** *For* $\tau \in (0,1)$, $\frac{\partial p^*}{\partial \theta} < 0$, *while for* $\tau \in (1,4)$, $\frac{\partial p^*}{\partial \theta} > 0$.
    - ***Proposition 1 implies that for sufficiently large values of τ, greater product homogeneity results in higher equilibrium price!***



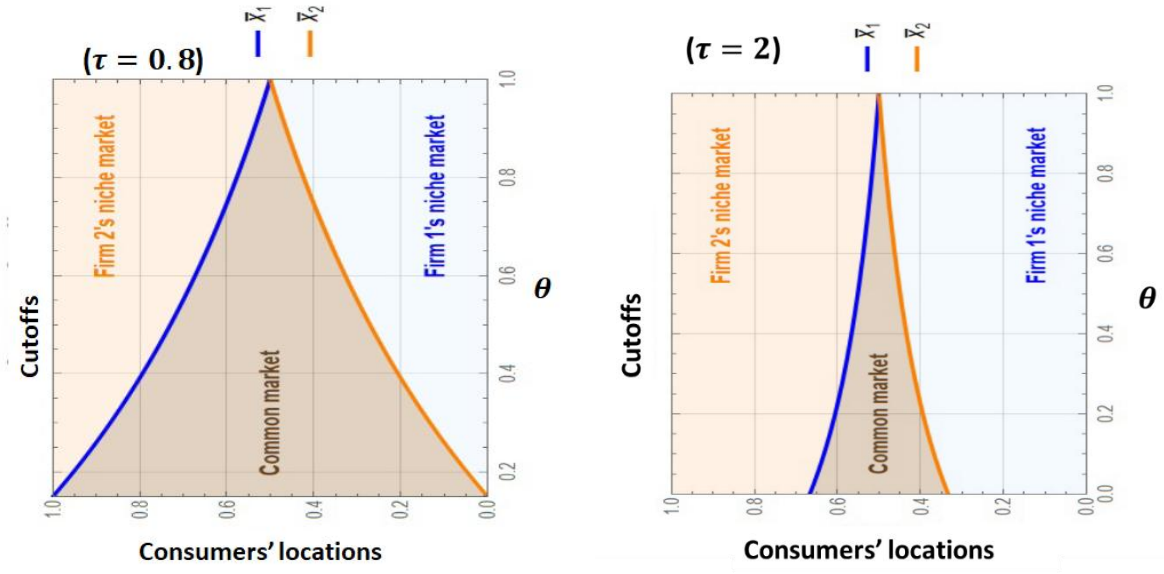
Equilibrium price for $\tau = 0.8$



Equilibrium price for $\tau = 1.2$

- This can be explained by the fact that consumers who are far away from firm 2 will decide to purchase its product only if it is substantially different from that of firm 1
- Increased product homogeneity turns those consumers unwilling to pay the transport cost, therefore
  - ***Increased product homogeneity increases firm 1's niche market***

- **Proposition 2:** As θ increases, in equilibrium both $\hat{x}_1, \hat{x}_2$, move inwards (towards the center). **Illustration of Proposition 2:**



- An increase of θ has two effects on equilibrium price:
  - Direct effect: price decreases
  - Indirect effect through the $\hat{x}_i$: price increases

- Despite the fact that an increase in θ may increase price, it turns out to always decrease profits
    - This happens because the direct quantity reduction in the common market is more important than the profit increase from the niche markets.
    - This result can be reversed if consumer density is not uniform across the H's line.
        - If there is a zone of low density around the center, the price increase due to higher θ may result in higher profits
- Consumer surplus decreases with θ for
    - a) utility is lower as product become closer substitutes and
    - b) prices go up.

E. CONCLUSIONS

- In a Hotelling model we relax the assumptions that the consumer buys only one unit from a single firm: we allow for
    - imperfect substitutability and
    - purchases in various quantities depending on price.
- Within this setting we show that greater inherent substitutability may lead to niche markets creation and a higher price for all the consumers.
- Despite the higher price, an increase in θ reduces profits, due to a reduction in willingness-to-pay in the common market.
    - However, this result can be reversed if consumer density is thinner around the center

- Our model bears some similarities with the cross-hauling result in International Trade (Brander & Spencer, 1986) where two rival firms in different countries cross-export to each other's (niche) market. Choosing to cross-haul is a prisoner's dilemma situation that reduces firms' profits.
    - In our model firms could agree to separate the market in two parts from the center.
- Differences between the models:
    - In B&S there is no common market, firms can identify their niche markets and price discrimination is feasible up to the cost of arbitrageurs
        - In our model, firms cannot identify each consumers' location and price discrimination is impossible.
        - The extent of cross-hauling is endogenous and depends on the inherent substitutability of the two firms' products.

THANKS FOR YOUR ATTENTION