ESSAY I: STYLIZED FACTS OF DEBT-TO-GDP OVER THE BUSINESS CYCLE

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Stylized facts of debt-to-GDP over the Business Cycle

- Over the business cycle (Recessions vs Expansions)
- In the EU (Core vs Periphery vs Non EU)
- Quantify the impact of each debt-to-GDP driver to its movement

Usefulness

- Analysis documents systematic comovements/empirical patterns/do not correspond to causal relationships but still informative
- Counterfactual experiments based on accounting schemes (this essay)
- Empirical research (dynamic panel regressions), e.g., on the differences and similarities in the response of interest rates and debt-to-GDP across the EU to common policy changes (ESM) and common policy responses (longer maturity debt) – ESSAY II (dynamic panel regressions and SVAR)

Data

Quarterly
Sample period: 2002Q1 to 2023Q4
12 Countries (Core: Belgium, France, Germany; Periphery: Greece, Ireland, Italy, Spain, Portugal; and NonEU: Canada, Japan, UK, USA)
Gross debt: Maastricht debt (Fase value) and SNA08 debt (Market price)
Databases: OECD, Eurostat, IMF

Business cycle episodes

1. Expansion (pre Great Recession)
2. Great Recession (GR)
3. Expansion (post Great Recession)
4. Covid Recession
5. Expansion (post Covid Recession)

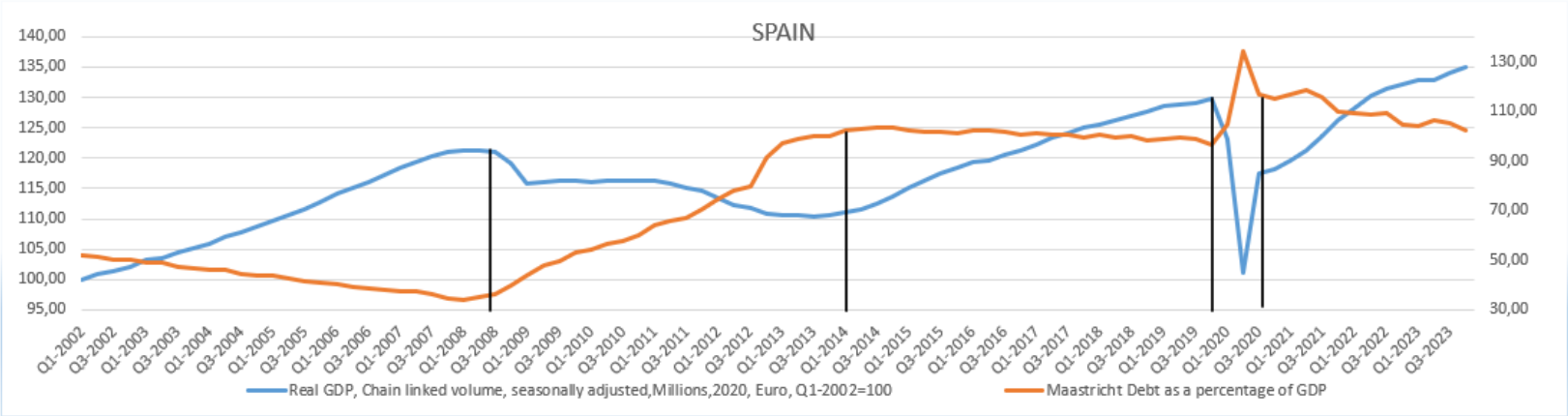
Country-specific business cycle episodes

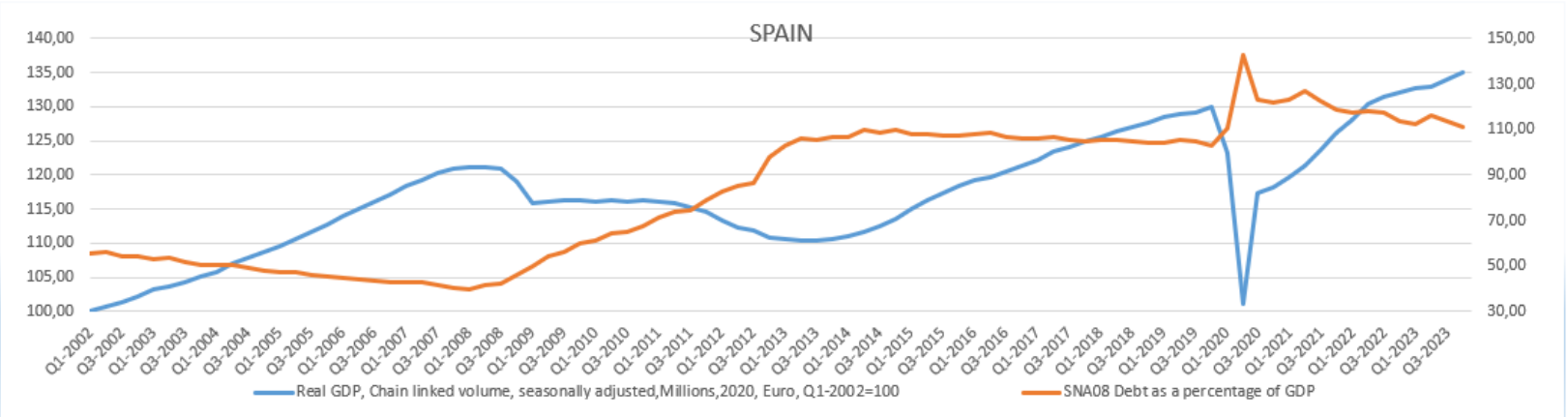
Table 1. Chronology of Expansions and Recessions (HP Filter,lamda=1600)

	Pre-Great Recession Expansion	Great Recession	Post-Great Recession Expansion	Covid Recession	Post-Covid Expansion
BELGIUM	Q3/2006-Q3/2008	Q4/2008-Q2/2014	Q3/2014-Q4/2019	Q1/2020-Q2/2021	Q3/2021-Q3-2023
CANADA	Q3/2005-Q4/2008	Q1/2009-Q4/2016	Q1/2017-Q4/2019	Q1/2020-Q3/2021	Q4/2021-Q3/2023
FRANCE	Q1/2006-Q3/2008	Q4/2008-Q4/2016	Q1/2017-Q4/2019	Q1/2020-Q2/2021	Q3/2021-Q4/2023
GERMANY	Q2/2006-Q4/2008	Q1/2009-Q4/2016	Q1/2017-Q1/2020	Q2/2020-Q4/2021	Q1/2022-Q3/2023
GREECE	Q1/2006-Q2/2010	Q3/2010-Q2/2014	Q3/2014-Q1/2020	Q2/2020-Q2/2021	Q3/2021-Q4/2023
IRELAND	Q4/2005-Q3/2008	Q4/2008-Q4/2014	Q1/2015-Q3/2018	Q4/2018-Q4/2020	Q1/2021-Q1/2023
ITALY	Q2/2006-Q3/2008	Q4/2008-Q4/2015	Q1/2016-Q4/2019	Q1/2020-Q2/2021	Q3/2021-Q4/2023
PORTUGAL	Q4/2006-Q4/2008	Q1/2009-Q2/2016	Q3/2016-Q4/2019	Q1/2020-Q4/2021	Q1/2022-Q4/2023
SPAIN	Q1/2006-Q4-2008	Q1/2009-Q2-2015	Q3/2015-Q4/2019	Q1/2020-Q1/2022	Q2/2022-Q4-2023
JAPAN	Q2/2005-Q3/2008	Q4/2008-Q4/2016	Q1/2017-Q3/2019	Q4/2019-Q3/2022	Q4/2022-Q4/2023
UNITED KINGDOM	Q3/2005-Q3/2008	Q4/2008-Q1/2014	Q2/2014-Q1/2020	Q2/2020-Q2/2021	Q3/2021-Q3/2023
UNITED STATES	Q4/2004-Q3/2008	Q4/2008-Q2/2017	Q3/2017-Q4/2019	Q1/2020-Q1/2021	Q2/2021-Q4/2023

Motivation

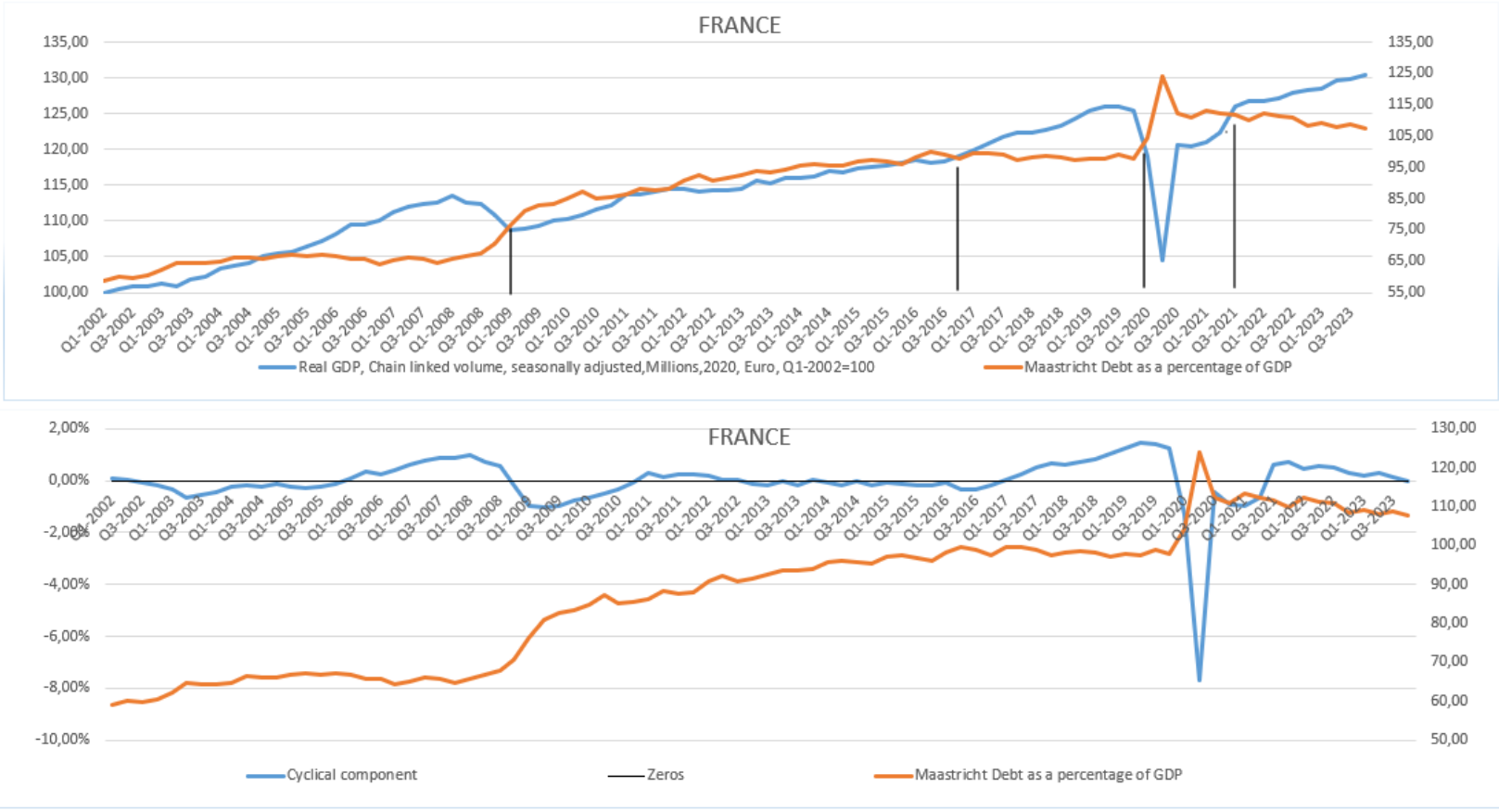
Figure 1. Debt-to-GDP and Real GDP – Spain (Maastricht and SNA08 definitions)





Motivation

Figure 2. Debt-to-GDP and Real GDP – France (Maastricht and SNA08 definitions)



I.Stylized Facts of Debt-to-GDP-Levels

Table 2. GDP changes (Quarterly data, Cumulative percentage changes, Maastricht definition)

Country/ Group of Countries	2002Q1-2008Q3	2008Q4-2012Q4	2013Q1-2019Q4	2020Q1-2021Q2	2021Q3-2023Q4
Belgium	15,84%	4,85%	12,58%	1,98%	4,62%
Canada	16,67%	5,75%	14,83%	0,48%	5,73%
France	12,34%	2,90%	9,49%	2,82%	3,48%
Germany	9,77%	4,20%	12,73%	1,71%	0,48%
Greece	27,11%	-26,32%	6,78%	-0,82%	7,65%
Ireland	25,47%	-5,70%	81,57%	17,36%	-1,11%
Italy	4,77%	-4,57%	4,79%	4,42%	4,87%
Japan	8,04%	1,78%	3,62%	-0,02%	2,03%
Portugal	5,42%	-8,00%	16,20%	-1,01%	8,55%
Spain	20,98%	-6,88%	17,42%	-1,54%	9,23%
United Kingdom	14,61%	2,90%	15,67%	0,50%	2,87%
United States	17,27%	6,09%	18,81%	3,53%	6,55%

I.Stylized Facts of Debt-to-GDP-Levels

Table 3. Debt-to-GDP ratio (Quarterly data, Cumulative percentage changes, Maastricht definition)

Country/ Group of Countries	2002Q1-2008Q3	2008Q4-2012Q4	2013Q1-2019Q4	2020Q1-2021Q2	2021Q3-2023Q4
Belgium	-18,33%	12,00%	-10,59%	10,25%	-7,16%
Canada	-23,29%	28,20%	2,43%	24,00%	-8,46%
France	13,51%	31,40%	5,07%	12,94%	-4,78%
Germany	8,85%	22,47%	-25,94%	14,08%	-7,65%
Greece	0,27%	47,97%	9,42%	15,30%	-20,53%
Ireland	13,26%	179,27%	-54,59%	-1,14%	-21,18%
Italy	-5,83%	19,00%	3,34%	12,15%	-10,76%
Japan	-	29,57%	5,23%	9,05%	-0,73%
Portugal	30,01%	70,35%	-11,41%	12,47%	-23,60%
Spain	-31,72%	126,11%	0,62%	19,57%	-11,97%
United Kingdom	36,22%	67,13%	3,52%	25,81%	-7,14%
United States	26,03%	43,36%	3,63%	15,69%	-1,92%

I.Stylized Facts of Debt-to-GDP-Levels

Table 4. Debt-to-GDP ratio (Quarterly data, Cumulative percentage changes, SNA08 definition)

Country/ Group of Countries	2002Q1-2008Q3	2008Q4-2012Q4	2013Q1-2019Q4	2020Q1-2021Q2	2021Q3-2023Q4
Belgium	-17,08%	11,79%	0,37%	-3,25%	7,31%
Canada	-20,65%	21,81%	3,23%	-1,16%	16,99%
France	11,24%	27,30%	15,05%	-0,44%	9,96%
Germany	8,80%	22,39%	-19,80%	-4,45%	13,50%
Greece	3,94%	47,47%	15,06%	1,18%	9,19%
Ireland	14,86%	168,85%	-30,09%	-1,01%	-11,81%
Italy	-5,78%	19,00%	12,35%	1,18%	7,13%
Japan	-	28,30%	15,31%	1,84%	10,63%
Portugal	20,93%	69,47%	20,38%	-5,78%	4,32%
Spain	-26,18%	111,43%	50,63%	0,00%	18,90%
United Kingdom	16,81%	27,86%	11,19%	-1,22%	16,18%
United States	25,03%	29,61%	7,32%	-0,59%	7,73%

I.Stylized Facts of Debt-to-GDP-Levels

- GDP gradually grows in general with fluctuations
- Debt-to-GDP also grows with fluctuations
- Debt-to-GDP does not in general mirror the movements of the denominator:Debt-to-GDP increases relatively sharply during the Great Recessions and the Covid Recessions but had a different behavior across countries in the 3 expansions (decreased in 19/36 cases in Maastricht and 6/36 in SNA08)
- except for Belgium all debt-to-GDP series are non-stationary and some are seasonal
- The behavior of Maastricht is in general not different than that of SNA08 debt.

II.Sources/Drivers of Debt-to-GDP-changes -Accounting and Theory

- Dynamic government budget constraint (Bohn (1992), Hall and Sargent (2011), etc.)

$$B_t - B_{t-1} \equiv D_t \tag{1}$$

- The cash-accruals accounting identity (Eisner and Pieper (1984), Fiorito (2014), Agoraki et al. (2021))

II.Sources/Drivers of Debt-to-GDP-changes -Accounting and Theory

$$D_t^a \equiv B_t - B_{t-1} - D_t + M_t - M_{t-1} \tag{2}$$

$$D_t \equiv IP_t + D_t^p \tag{3}$$

$$i_t^e \equiv \frac{IP_t}{B_{t-1}} \tag{4}$$

$$S_t = M_t - M_{t-1} \tag{5}$$

$$B_t \equiv (1 + i_t^e)B_{t-1} + D_t^p + D_t^a - S_t \tag{6}$$

$$b_t \equiv \frac{(1 + i_t^e)}{(1 + \pi_t)(1 + g_t^y)} b_{t-1} + d_t^p + d_t^a - s_t \tag{7}$$

$$g_t^b \equiv b_t - b_{t-1} \simeq i_t^e b_{t-1} - \pi_t b_{t-1} - g_t^y b_{t-1} + d_t^p + d_t^a - s_t \tag{8}$$

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II.Stylized Facts of Debt-to-GDP of the whole sample

Table 5: Correlation: Debt-to-GDP changes and GDP growth

	Cor {b(t) , dGDP(t-2)}	Cor {b(t) , dGDP(t-1)}	Cor {b(t) , dGDP(t)}	Cor {b(t), dGDP(t+1)}	Cor {b(t) , dGDP(t+2)}
BELGIUM	0,050	-0,125	**-0,380**	0,123	0,217
CANADA	-0,173	-0,225	**-0,694**	0,277	0,136
FRANCE	0,003	-0,297	**-0,359**	0,334	0,160
GERMANY	-0,013	-0,326	**-0,325**	0,309	0,115
GREECE	-0,305	-0,161	**-0,361**	0,001	-0,022
IRELAND	-0,156	-0,255	**-0,313**	-0,126	-0,177
ITALY	-0,131	**-0,375**	-0,290	0,252	0,146
JAPAN	**-0,447**	-0,275	-0,340	0,393	0,111
PORTUGAL	-0,133	-0,224	**-0,227**	-0,027	0,029
SPAIN	-0,161	-0,275	**-0,374**	0,015	0,026
UNIT.KINGDOM	-0,195	-0,189	**-0,375**	0,311	-0,037
USA	-0,150	-0,170	**-0,475**	0,193	0,007

- In all countries D-t-G changes are negatively correlated with real GDP growth. (12/12).
- 10/12 are negatively and contemporaneous correlated

II.Stylized Facts of Debt-to-GDP of the whole sample

Table 6: Correlation: Debt-to-GDP changes and Inflation

	Cor {dbt , dπ(t-2)}	Cor {dbt , dπ(t-1)}	Cor {dbt , dπt}	Cor {dbt , dπ(t+1)}	Cor {dbt , dπ(t+2)}
BELGIUM	-0,074	-0,090	-0,175	-0,162	-0,084
CANADA	-0,193	-0,355	**-0,390**	0,060	0,092
FRANCE	-0,196	0,029	0,171	**-0,417**	-0,210
GERMANY	-0,224	0,003	-0,167	**-0,332**	-0,099
GREECE	-0,134	-0,280	-0,042	-0,238	**-0,281**
IRELAND	-0,223	**-0,330**	-0,213	-0,327	-0,114
ITALY	-0,134	-0,180	-0,052	-0,191	**-0,269**
JAPAN	-0,068	-0,139	-0,162	**-0,379**	-0,234
PORTUGAL	-0,155	**-0,347**	-0,130	-0,298	-0,231
SPAIN	-0,340	-0,370	**-0,432**	-0,380	-0,385
UNIT.KINGDOM	0,082	-0,010	0,223	**-0,454**	-0,126
USA	-0,264	-0,273	**-0,416**	-0,211	-0,184

- D-t-G changes are predominantly negatively correlated with inflation across the sample.
- Periods of above-average D-t-G changes are followed by below-average inflation at the same or within one or two quarters later.
- There is significant cross-country heterogeneity regarding the lag at which the strongest relationship is observed.
- Only a few countries (CAN, SPA, USA) exhibit their strongest correlation contemporaneously.

II.Stylized Facts of Debt-to-GDP of the whole sample

Table 7: Correlation: Debt-to-GDP changes and Primary Balance

	Cor {bt , dp(t-2)}	Cor {bt , dp(t-1)}	Cor {bt , dp(t)}	Cor {bt, dp(t+1)}	Cor {bt , dp(t+2)}
BELGIUM	0,057	0,138	**0,340**	0,161	0,209
CANADA	0,098	0,203	**0,688**	0,558	0,412
FRANCE	-0,104	0,159	**0,442**	0,305	0,345
GERMANY	0,102	0,348	**0,434**	0,436	0,323
GREECE	0,194	0,234	**0,368**	0,282	0,235
IRELAND	0,344	0,346	**0,599**	0,450	0,496
ITALY	**-0,226**	-0,094	-0,004	0,068	0,114
JAPAN	0,275	0,461	0,605	**0,657**	0,632
PORTUGAL	0,437	0,408	0,493	**0,518**	0,502
SPAIN	0,564	0,665	**0,827**	0,691	0,631
UNIT.KINGDOM	0,221	0,369	**0,665**	0,493	0,458
USA	0,150	0,186	**0,521**	0,471	0,401

- The most frequent empirical pattern here is: higher(lower) D-t-G changes are most often associated with weaker (stronger)primary balances
- The timing pattern is very clear: the relationship is predominantly synchronous, with the strongest statistically significant correlation occurring contemporaneously in 9 out of 12 countries.

III.Stylized Facts of Debt-to-GDP of the whole sample

Table 8: Correlation: Debt-to-GDP changes and Effective Interest Rate

	Cor {b(t) , i(t-2)}	Cor {b(t) , i(t-1)}	Cor {b(t) , i(t)}	Cor {b(t), i(t+1)}	Cor {b(t) , i(t+2)}
BELGIUM	-0,024	-0,037	-0,012	-0,091	-0,061
CANADA	-0,092	-0,113	-0,138	-0,199	**-0,219**
FRANCE	0,143	0,133	0,105	0,042	0,026
GERMANY	**0,249**	0,237	0,230	0,195	0,197
GREECE	0,136	0,118	0,130	0,069	0,086
IRELAND	0,249	0,252	**0,282**	0,178	0,223
ITALY	0,056	0,044	0,029	-0,065	-0,087
JAPAN	**0,243**	0,205	0,182	0,126	0,064
PORTUGAL	**0,403**	0,402	0,413	0,333	0,331
SPAIN	0,038	0,017	-0,002	-0,064	-0,088
UNIT.KINGDOM	0,137	0,060	0,026	-0,043	-0,056
USA	**0,222**	0,197	0,164	0,081	0,062

- Overall, D-t-G changes and the effective interest rate appear to be largely uncorrelated, with no systematic pattern emerging across countries.
- Nevertheless, among the statistically significant cases, the association is predominantly positive (5/6), with Canada as the only negative exception.

III.Stylized Facts of Debt-to-GDP of the whole sample

Table 9: Correlation: Debt-to-GDP changes and Cash-accruals Discrepancy

Correlation (QUARTERLY) : Debt-to-GDP Changes (Maastricht) and CAD					
	Cor {b(t) , CAD(t-2)}	**Cor {b(t) , CAD(t-1)}**	**Cor {b(t) , CAD(t)}**	**Cor {b(t) , CAD(t+1)}**	**Cor {b(t) , CAD(t+2)}**
BELGIUM	0,145	**0,300**	0,258	0,206	0,214
CANADA	0,217	**0,665**	0,595	0,482	0,378
FRANCE	0,213	**0,567**	0,553	0,433	0,346
GERMANY	0,155	**0,456**	0,387	0,298	0,195
GREECE	-0,103	0,223	0,099	0,128	**0,292**
IRELAND	0,357	**0,480**	0,421	0,397	0,363
ITALY	0,045	0,373	**0,393**	0,349	0,346
JAPAN	-0,231	0,140	**0,282**	0,278	0,130
PORTUGAL	0,093	**0,443**	0,389	0,390	0,426
SPAIN	-0,186	0,023	0,127	0,117	0,075
UNIT.KINGDOM	0,346	0,658	**0,698**	0,665	0,586
USA	0,048	**0,501**	0,479	0,408	0,253

- A remarkably strong positive association emerges between debt-to-GDP changes and CAD, with correlation coefficients reaching 0.40 to 0.70 in several countries.
- The relationship displays a clear timing pattern: CAD leads debt-to-GDP changes by one quarter in 7 out of 12 countries.
- These findings underscore the quantitatively important role of stock-flow adjustments in shaping public debt dynamics beyond the conventional fiscal balance.

III.Stylized Facts of Debt-to-GDP of the whole sample

Table 10: Correlation: Debt-to-GDP changes and Seignorage

Correlation (QUARTERLY) : Debt-to-GDP Changes (Maastricht) and Seignorage					
	Cor {b(t) , S(t-2)}	**Cor {b(t) , S(t-1)}**	**Cor {b(t) , S(t)}**	**Cor {b(t) , S(t+1)}**	**Cor {b(t) , S(t+2)}**
BELGIUM	0,157	0,181	**0,211**	**0,211**	0,207
CANADA	-0,075	0,191	0,394	0,439	**0,462**
FRANCE	0,028	0,112	0,179	0,148	0,129
GERMANY	0,244	**0,265**	0,185	0,103	0,020
GREECE	0,090	0,079	0,045	0,062	0,125
IRELAND	**0,361**	0,338	0,313	0,288	0,267
ITALY	-0,131	0,020	0,023	0,044	0,017
JAPAN	-0,158	0,006	0,078	0,030	-0,012
PORTUGAL	0,064	0,202	0,291	0,309	**0,325**
SPAIN	**-0,218**	-0,150	-0,086	-0,106	-0,123
UNIT.KINGDOM	0,074	0,246	0,409	0,447	**0,490**
USA	-0,049	0,131	0,298	**0,307**	0,273

- Overall, no strong or universal correlation pattern emerges between debt-to-GDP changes and seigniorage, with statistically significant relationships observed in only 8 out of 12 countries.
- Among the statistically significant cases, the association is predominantly positive (7/8), with Spain as the only negative exception.
- The strength of the relationship is generally moderate, although relatively large positive correlations are observed in Canada (0.462) and the United Kingdom (0.490).
- The weak and heterogeneous pattern suggests that seigniorage does not constitute a systematic short-run driver of debt-to-GDP dynamics across countries, although it may play a more relevant country-specific role in certain cases.

IV.Summary of Stylized Facts over the Business Cycle

Table 11. Summary of Stylized Facts over the Business Cycle

Source	Expansion	Recession
Real GDP growth	Strong negative Mostly synchronous **Vs overall: slighly weaker**	Strong negative Mostly synchronous **Vs overall: greater in magnitude** Country-specific deviations
Inflation	Weak/moderate negative No systematic lead/lag **Vs overall: similar or slighly stronger**	Moderate negative More homogeneous than expansions **Vs overall: similar** Timing: often lag (+1)
Effective Interest Rate	No common pattern Mostly weak/insignicant **Vs overall: similar** Significant : Greece, Portugal, USA, GER, UK, IRE	No common pattern Mostly weak/insignicant **Vs overall: similar** Significant: IRE USA
Primary Deficit	Strong positive Mostly synchronous **Vs overall: some weaker some stronger**	Strong positive Highly homogeneous **Vs overall: Magnitude much stronger**
Cash-accruals discrepancy	Strong positive in all cases Mostly synchronous **Vs overall: Slightly weaker** Strong significant: CAN, UK, USA	Strong positive **Vs overall: Magnitude much stronger**
Seignorage	Weak/no systematic pattern **Vs overall: Similar** Significant: IRE, JAP, USA	Weak/no systematic pattern **Vs overall: Similar** Significant: IRE, CAN, UK, USA

Key findings across the business cycle

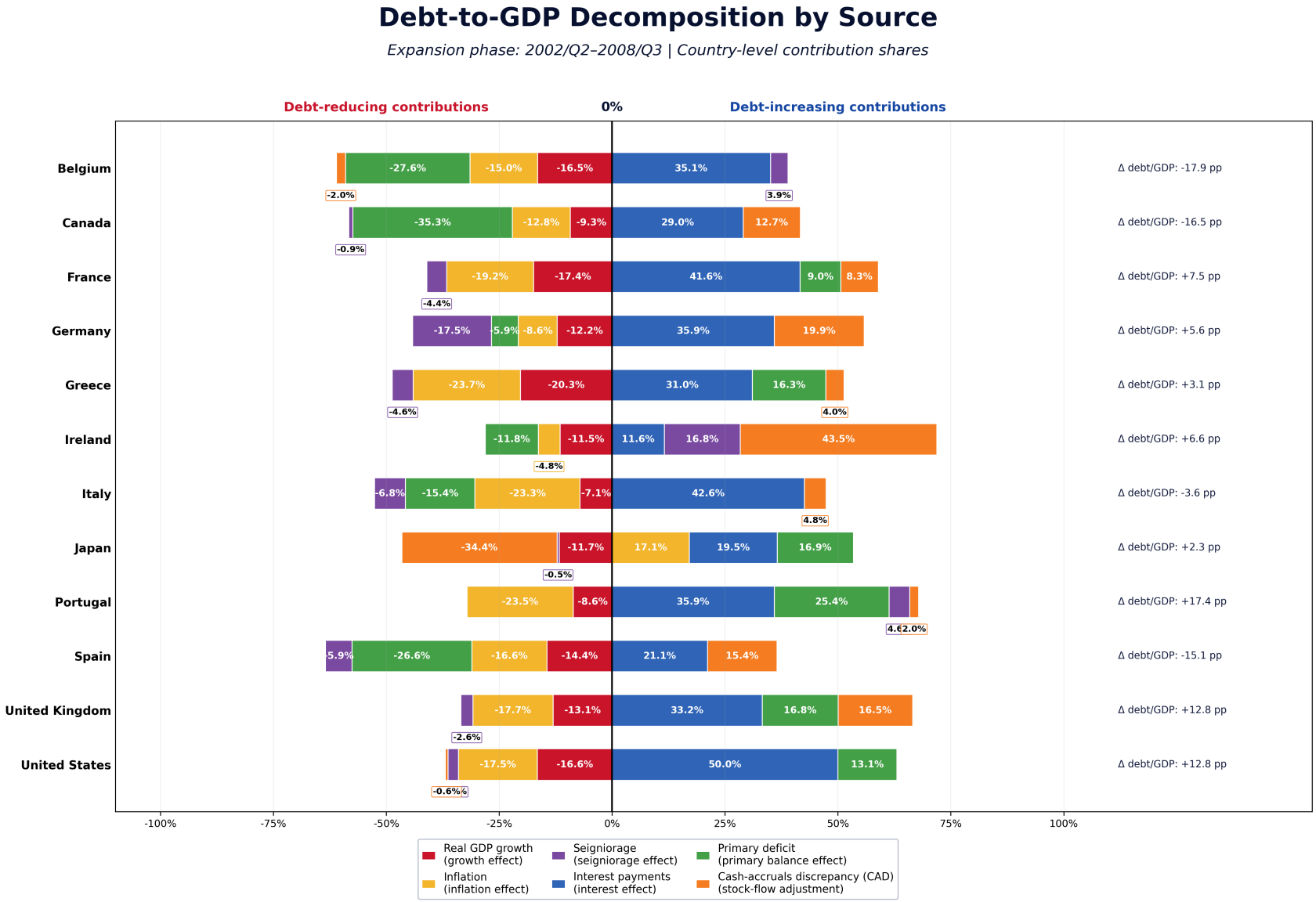
- The main stylized facts observed in the overall sample generally persist across business-cycle phases, particularly in terms of statistical significance and direction.
- Business-cycle conditions primarily affect the magnitude of the relationships, rather than their direction or timing.
- Expansions tend to amplify the relationship between debt dynamics and inflation.
- Recessions tend to amplify the relationship between debt dynamics and GDP growth, CAD and Primary Balance.
- The effective interest rate and seigniorage remain weak, mostly statistically insignificant, and highly heterogeneous, with no systematic pattern across business-cycle phases.
- Overall, separating expansions from recessions reveals that debt dynamics become stronger in magnitude with the main sources that affect them through the whole periods during downturns

Key findings across the business cycle

- Expansions: Real GDP growth is the most consistent source in terms of direction and magnitude.
- Recessions: The primary deficit is the most homogeneous source overall, across direction, timing, and magnitude.
- CAD in recessions: Highly homogeneous in direction (11 out of 12 significant correlations are positive), but heterogeneous in timing across countries.

V.Contributions of debt-to-GDP sources over the Business Cycle

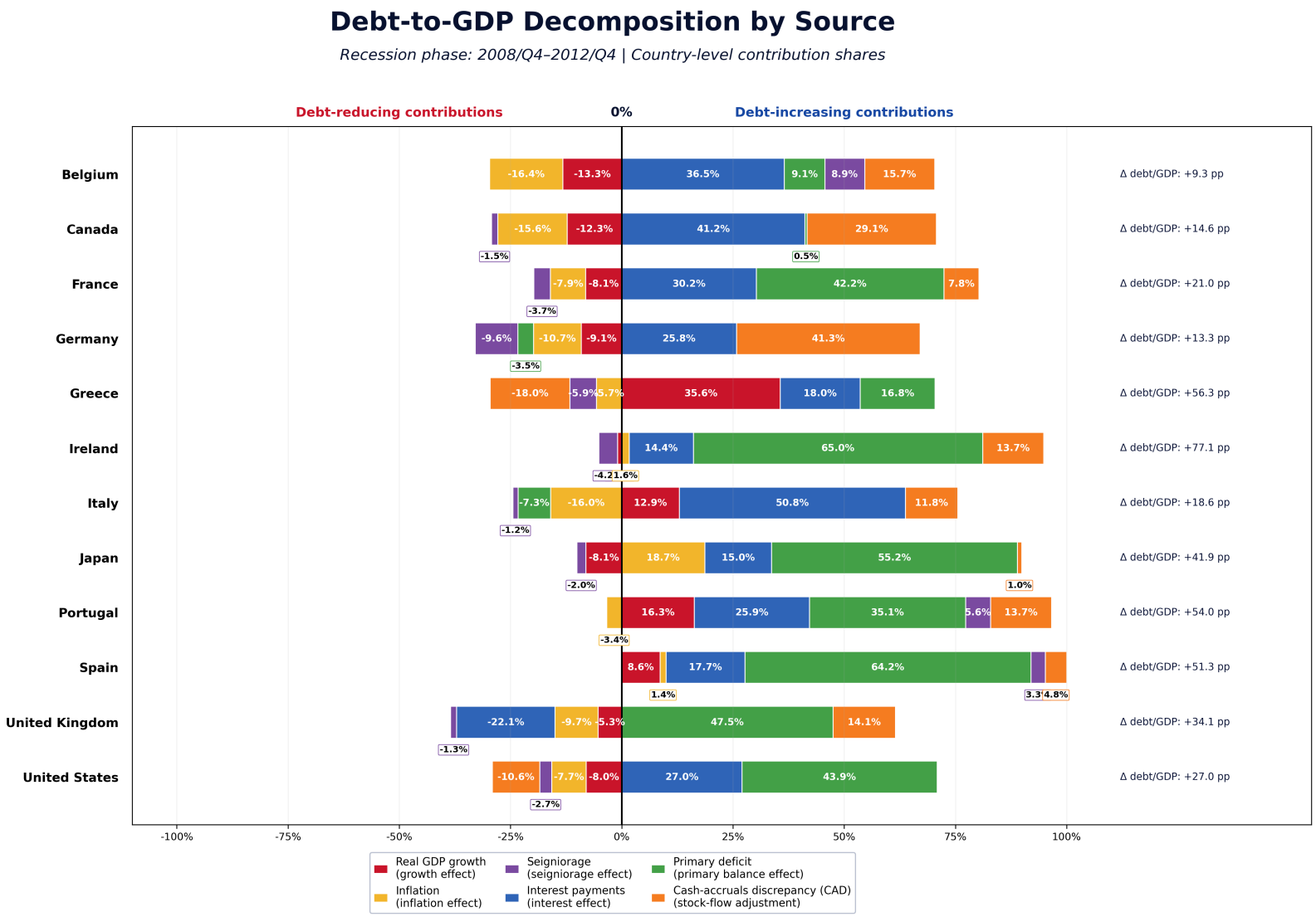
Figure 3: Debt decomposition – Expansion 2002/Q2–2008/Q3 (Maastricht definition)



Note: Segment lengths correspond exactly to the contribution percentages reported in the uploaded data. Small segments are labelled in white callout boxes so that every percentage is visible. Maastricnt debt definition.

V.Contributions of debt-to-GDP sources over the Business Cycle

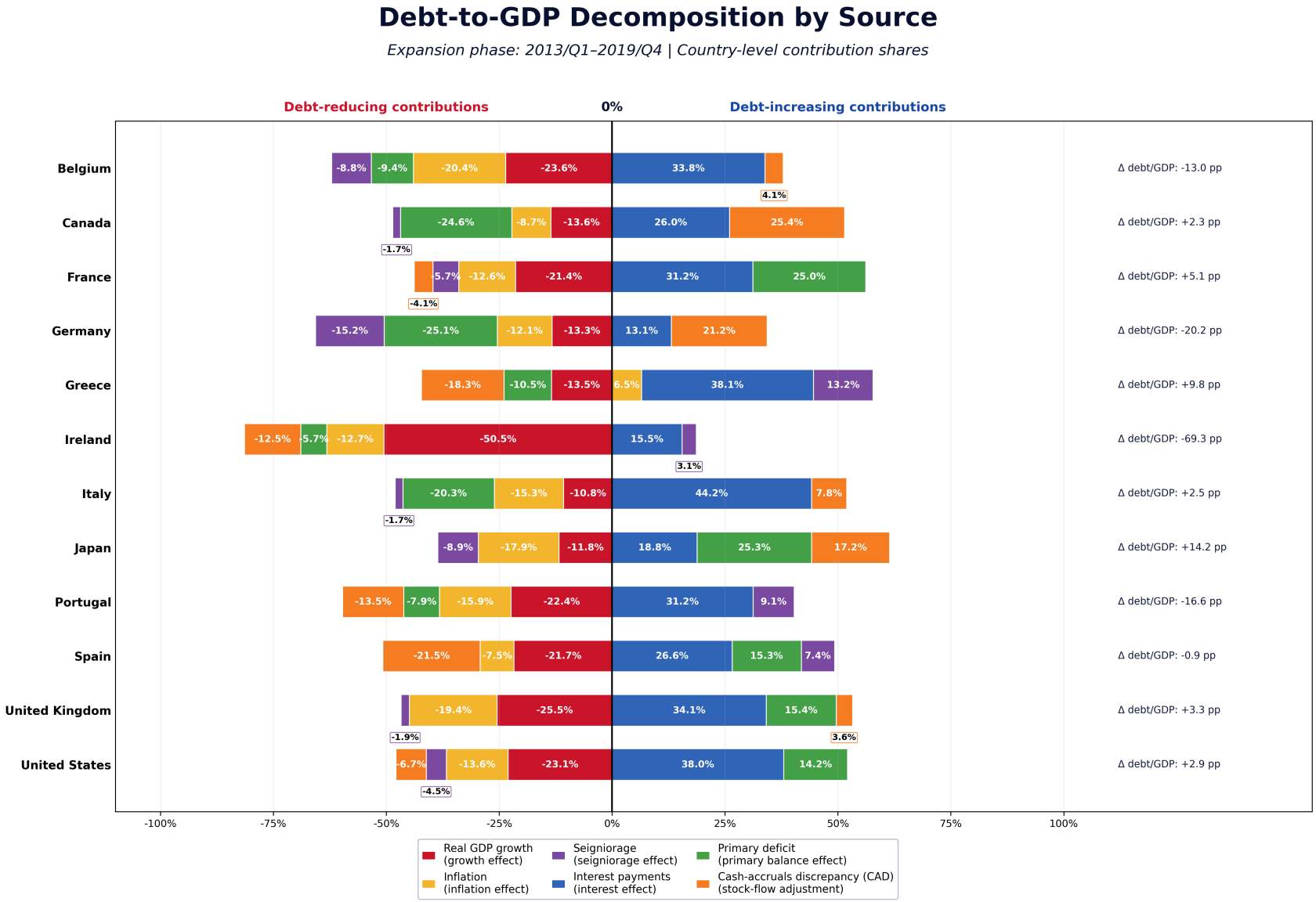
Figure 4: Debt decomposition – Recession 2008/Q4–2012/Q4 (Maastricht definition)



Note: Segment lengths correspond exactly to the contribution percentages reported in the uploaded data. Small segments are labelled in white callout boxes so that every percentage is visible. Maastricht debt definition.

V.Contributions of debt-to-GDP sources over the Business Cycle

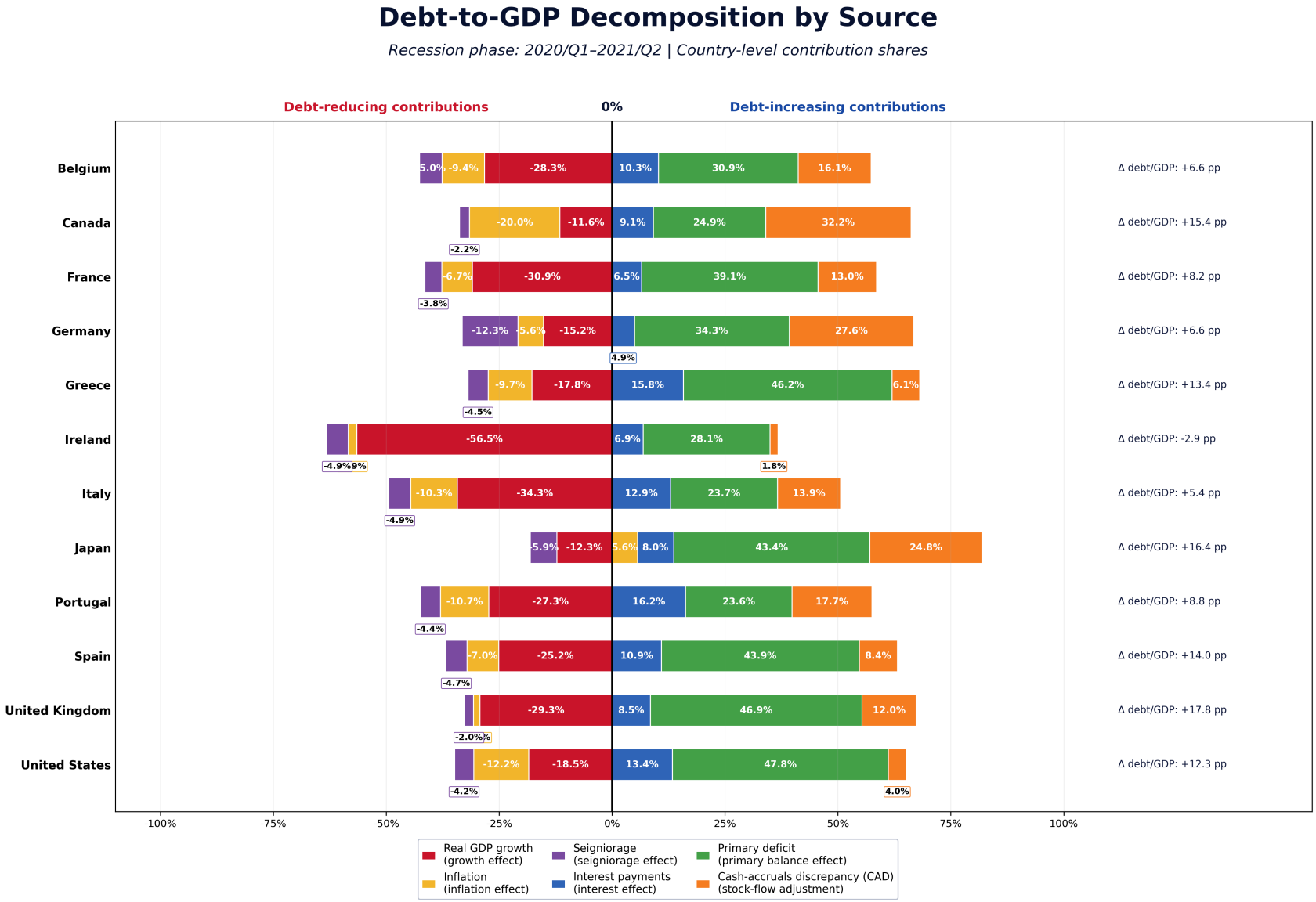
Figure 5: Debt decomposition – Expansion 2013/Q1–2019/Q4 (Maastricht definition)



Note: Segment lengths correspond exactly to the contribution percentages reported in the uploaded data. Small segments are labelled in white callout boxes so that every percentage is visible. Maastricht debt definition.

V.Contributions of debt-to-GDP sources over the Business Cycle

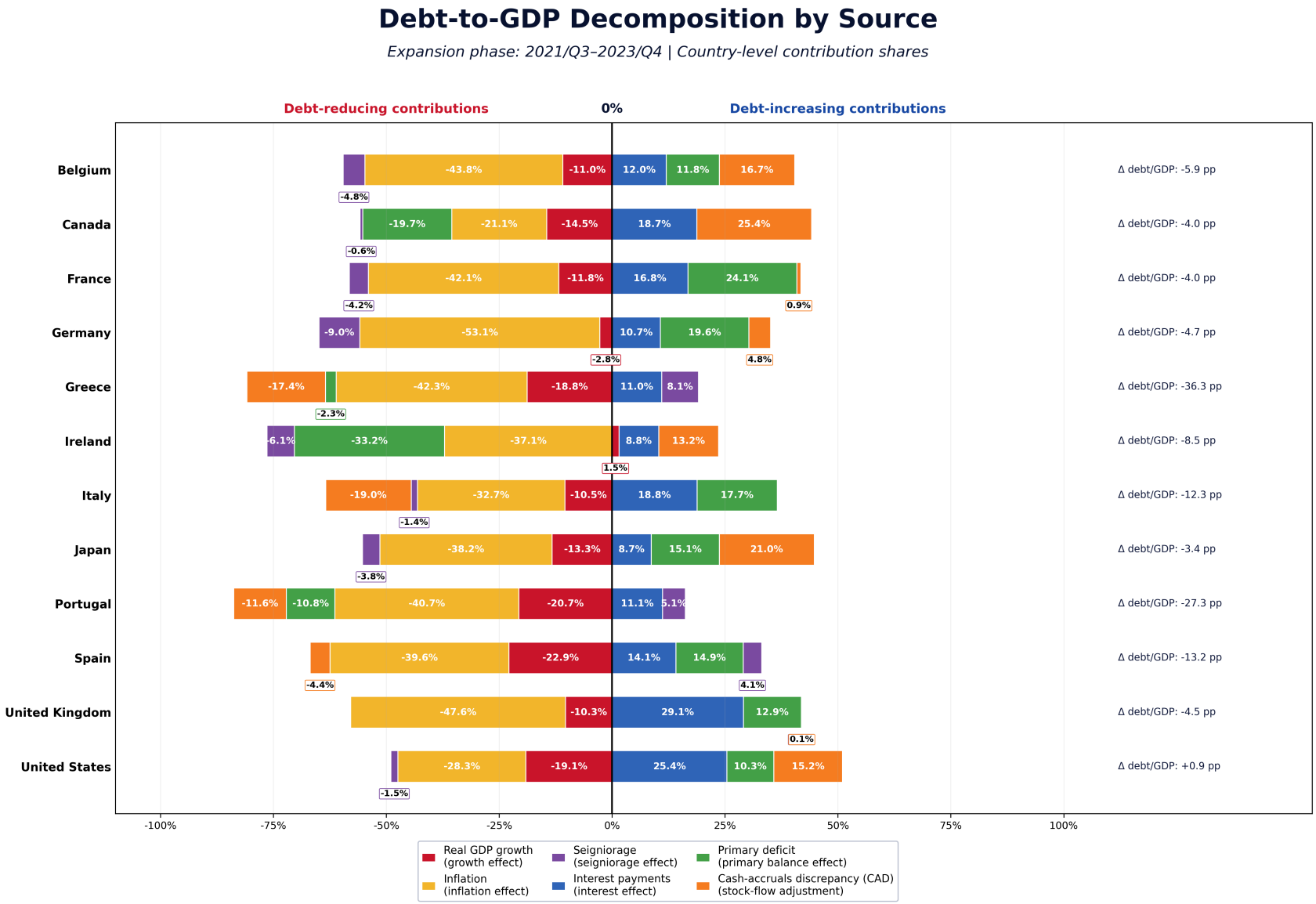
Figure 6: Debt decomposition – Recession 2020/Q1–2021/Q2 (Maastricht definition)



Note: Segment lengths correspond exactly to the contribution percentages reported in the uploaded data. Small segments are labelled in white callout boxes so that every percentage is visible. Maastricht debt definition.

V.Contributions of debt-to-GDP sources over the Business Cycle

Figure 7: Debt decomposition – Expansion 2021/Q3–2023/Q4 (Maastricht definition)



Note: Segment lengths correspond exactly to the contribution percentages reported in the uploaded data. Small segments are labelled in white callout boxes so that every percentage is visible. Maastricht debt definition.

Conclusions

- Debt-to-GDP dynamics display clear empirical regularities, although their strength and timing vary across sources, countries, and business-cycle phases.
- Real GDP growth exhibits the most robust macroeconomic pattern, with an almost universally negative and predominantly synchronous association with debt-to-GDP changes, while primary deficits and cash-accruals discrepancies emerge as the strongest positive correlates.
- The contribution analysis reveals a marked asymmetry across the business cycle: favorable macroeconomic conditions generally support debt reduction during expansions, whereas primary deficits become a dominant debt-increasing force during recessions.
- Overall, public debt dynamics cannot be understood through fiscal balances alone: macroeconomic conditions and stock-flow adjustments are quantitatively important components of debt-to-GDP dynamics.