Wage and Unemployment Dynamics under Incomplete Employment Contracts

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Introduction

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  - **Staggered adjustments (Calvo)**: Gertler and Trigari (2009); Blanco et al. (2025).
  - **Complete contracts with exogenous adjustment frictions**: (Blanco and Drenik, 2023; Jo, 2025; Elsby et al., 2025).

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  - Develop a benchmark GE framework under incomplete contracts with **heterogeneous agents**, as well as endogenous hiring and separations; (NEW!)

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  - **Aggregate and distributional consequences** of the model inherent features for wage, output, and unemployment dynamics. (NEW!)

What we do (1)

- Optimal wage setting under incomplete employment contracts (partial equilibrium):
  - Building on Elsby (2009) and Fongoni (2024);
  - Workers' effort is discretionary and not contractible; **asymmetric effort response (reciprocity)** to nominal wage changes;

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- Calibration to match empirical regularities on **wage dynamics, rent sharing and labor share**: promising for "meaningful" quantitative counterfactual analyses.

What we do (2)

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  - GE model + random search + endogenous separations + idiosyncratic shocks;
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  - Baseline: labor-only, no assets/capital accumulation.
- Contributions:
  - Provide a benchmark macroeconomic framework of wage and unemployment dynamics under incomplete employment contracts;
  - Aggregate and distributional implications of state-dependent & path-dependent wage rigidity: still work in progress…

Wage Setting Model (Partial Equilibrium)

General model environment

- Firm-worker employment relationship ("match") under complete and perfect info:
  - Firm's output Y_t is endogenous to worker's effort f: $Y_t = Z_t f(W_t, R_t)$.

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  - Matches separate at the exogenous rate $\delta > 0$, as well as endogenously;
  - Exogenous match productivity Z_t evolves according to a GBM with drift:

$$dZ_t = (\mu + \pi) Z_t dt + \sigma Z_t dB_t.$$

Worker effort and reference wage adaptation

- Worker's effort governed by an *asymmetric reference-dependent effort function*:

$$f(W_t, R_t) = \frac{W_t^{1-\gamma}}{1-\gamma} + \frac{\eta}{1-\gamma} \begin{cases} W_t^{1-\gamma} - R_t^{1-\gamma}, & \text{if } W_t > R_t, \\ \lambda\left(W_t^{1-\gamma} - R_t^{1-\gamma}\right), & \text{if } W_t < R_t, \end{cases}$$

where $\eta > 0$; $\gamma \in (0, 1)$; and $\lambda \geq 1$ determines the relative strength of effort to wage changes below the reference wage. *If* $\lambda > 1 \rightarrow$ *wage rigidity*.

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▶ Given an initial R_s, the worker's reference wage R_t evolves according to

$$dR_t = (W_t - R_{t^-})dN_t.$$

where $\{N_t\}_{t \geq 0}$ is a standard Poisson point process with constant intensity $\nu > 0$, independent of the Brownian motion $B_t \rightarrow$ *discrete jump process*.

Firm's problem and recursive formulation

- The firm seeks to maximize its expected PD value of profits by choosing an admissible sequence of wages $\{W_t\}$ and a stopping/layoff time $\tau < \infty$:

$$\mathcal{J}(R, Z) = \sup_{\substack{W \in \mathcal{W} \\ \tau \in \mathcal{T}}} \mathbb{E}\left[\int_s^{\widetilde{\tau}} e^{-\hat{\rho}(t-s)} \, \Pi(W_t, R_t, Z_t) dt \,|\, \mathcal{F}_s \right],$$

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- Value function away from layoff $\rightarrow$ Hamilton-Jacobi-Bellman (HJB) equation:

$$\hat{\rho}\mathcal{J}(R, Z) = \sup_{W} \left\{ \Pi(W, R, Z) + (\mu + \pi) Z \mathcal{J}_Z + \frac{1}{2}\sigma^2 Z^2 \mathcal{J}_{ZZ} + \nu \left(\mathcal{J}(W, Z) - \mathcal{J}(R, Z) \right) \right\}$$

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- Global value function with optimal stopping $\rightarrow$ HJB Variational Inequality:

$$\max \left\{ \sup_W \{\Pi(W, R, Z) + \nu \left(\mathcal{J}(W, Z) - \mathcal{J}(R, Z) \right)\} + \mathcal{L}\mathcal{J}(R, Z) - \hat{\rho}\mathcal{J}(R, Z), -\mathcal{J}(R, Z) \right\} = 0$$

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- Transversality condition: $\limsup_{T \to \infty} \mathbb{E}\left[e^{-\hat{\rho}T} \mathcal{J}(R_T, Z_T) | \mathcal{F}_s \right] = 0$

First-order conditions for the (interior) optimal wage

- The firm sets the wage to optimize the sum of the *immediate payoff* as well as the *expected continuation value*:

$$\underbrace{Z_t f_W(W,R;\lambda)}_{\substack{\text{current}\\ \text{marginal benefit}}} - \underbrace{1}_{\substack{\text{current}\\ \text{marginal cost}}} + \underbrace{\nu \mathcal{J}_R(W,Z)}_{\substack{\text{expected future}\\ \text{marginal cost}}} = 0, \text{ for } W \neq R,$$

where $f_W(W,R;\lambda) \triangleq \frac{\partial f(W,R;\lambda)}{\partial W}$, $\mathcal{J}_R(W,Z) \triangleq \frac{\partial J(R',Z)}{\partial R}\big|_{W=R'}$.

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- This translates to:

$$\widetilde{W}(R,Z) = \left(\frac{Z(1+\eta)}{1-\nu\mathcal{J}_R(\widetilde{W},Z)}\right)^{1/\gamma}, \quad \text{if } \widetilde{W} > R, \tag{1}$$

$$\widetilde{W}(R,Z) = \left(\frac{Z(1+\lambda\eta)}{1-\nu\mathcal{J}_R(\widetilde{W},Z)}\right)^{1/\gamma}, \quad \text{if } \widetilde{W} < R. \tag{2}$$

Overview of solution method

- Establish the existence of a *non-trivial inaction region* if and only if worker's reference-dependent reciprocity is asymmetric ($\lambda > 1$).

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- Construct the candidate value function and go through a verification theorem.
- Candidate value $=$ unique viscosity solution to HJB $=$ true value function. ✓

Optimal wage policy (if the worker's reservation wage never binds)

The optimal wage policy for every $t \geq 0$ *is*

$$\widetilde{W}(R_t, Z_t) = \begin{cases} (Z_t/U)^{1/\gamma} > R_t, & \text{if } Z_t > UR^\gamma \quad \text{(raise)} \\ R_t, & \text{if } Z_t \in [LR_t^\gamma UR^\gamma] \quad \text{(freeze)} \\ (Z_t/L)^{1/\gamma} < R_t, & \text{if } Z_t \in (\underline{L}R_t^\gamma, LR_t^\gamma) \quad \text{(cut)} \\ \text{layoff} & \text{if } Z_t \leq \underline{L}R_t^\gamma \end{cases}$$

where $U > L > \underline{L}$ *are real constants determined by the smooth-conditions.*

▷ State- and path-dependent "Ss" policy characterized by time-varying thresholds.

Optimal wage policy (if the worker's reservation wage may bind)

The constrained-optimal wage policy for every $t \geq 0$ is

$$\widetilde{W}(R_t, Z_t) = \begin{cases} (Z_t/U)^{1/\gamma} > R_t, & \text{if } Z_t > U R^{\gamma} \quad \text{(raise)} \\ R_t, & \text{if } Z_t \in [L R_t^{\gamma} U R^{\gamma}] \quad \text{(freeze)} \\ (Z_t/L)^{1/\gamma} < R_t, & \text{if } Z_t \in [L^q R_t^{\gamma}, L R_t^{\gamma}) \quad \text{(cut)} \\ \underline{W}_t(R_t, Z_t), & \text{if } Z_t \in (\underline{L}^c R_t^{\gamma}, L^q R_t^{\gamma}) \quad \text{(pay reservation wage)} \\ \textit{layoff} & \text{if } Z_t \leq \underline{L}^c R_t^{\gamma} \end{cases}$$

where $U > L > \underline{L}^c$ are real constants determined by the smooth-conditions (numerically different to those of the previous slide), and L^q is determined by the worker's problem.

Firm's value function

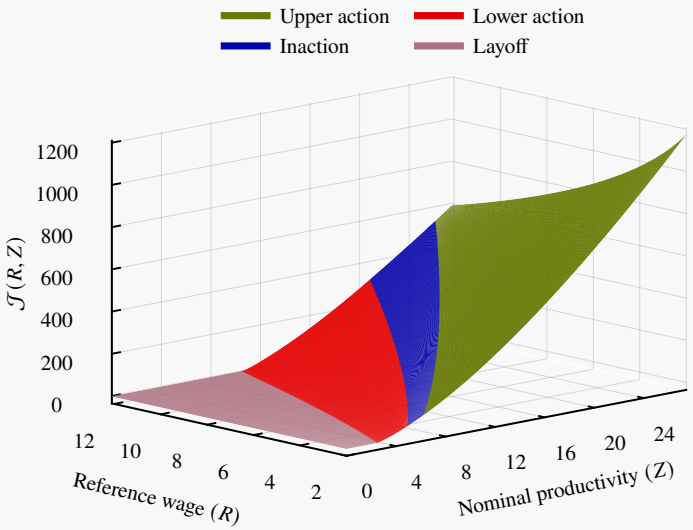
$$\mathcal{J}(R,Z)=\begin{cases}\mathcal{J}^U(R,Z;\widetilde{W}), & \text{if } Z_t > UR^\gamma \\ \overline{\mathcal{J}}(R,Z), & \text{if } Z_t \in [LR^\gamma, UR^\gamma], \\ \mathcal{J}^L(R,Z;\widetilde{W}), & \text{if } Z_t \in (\underline{L}R^\gamma, LR^\gamma), \\ 0 \text{ (layoff)} & \text{if } Z_t \leq \underline{L}R^\gamma. \end{cases}$$

$$\mathcal{J}^U(R,Z)=A_UZ^{1/\gamma}-B_UZR^{1-\gamma}+\Psi_UZ^{\check{\beta}_2}R^{1-\gamma\check{\beta}_2},$$

$$\overline{\mathcal{J}}(R,Z)=\frac{ZR^{1-\gamma}}{(1-\gamma)(\rho+\delta-\mu)}-\frac{R}{\rho+\delta+\pi}+\Omega_1Z^{\beta_1}R^{1-\gamma\beta_1}+\Omega_2Z^{\beta_2}R^{1-\gamma\beta_2},$$

$$\mathcal{J}^L(R,Z)=A_LZ^{1/\gamma}-B_LZR^{1-\gamma}+\Psi_{1L}Z^{\check{\beta}_1}R^{1-\gamma\check{\beta}_1}+\Psi_{2L}Z^{\check{\beta}_2}R^{1-\gamma\check{\beta}_2}.$$

3D graph of the value function



Wage Dynamics (Partial Equilibrium)

Explicit wage dynamics

- Optimal wage policy in the action regions ($Z_t \notin [LR^\gamma, UR^\gamma]$ and $\widetilde{W}_t > \underline{W}_t(R, Z)$) follows a geometric Brownian motion with modified drift and volatility

$$\frac{d\widetilde{W}_t}{\widetilde{W}_t} = \left(\frac{\mu + \pi}{\gamma} + \frac{1}{2}\frac{\sigma^2(1-\gamma)}{\gamma^2}\right)dt + \frac{\sigma}{\gamma}dB_t,$$

$$\widetilde{W}_t = \widetilde{W}_{\tau_i} \exp\left\{\frac{1}{\gamma}\left(\mu + \pi - \frac{\sigma^2}{2}\right)(t - \tau_i) + \frac{\sigma}{\gamma}(B_t - B_{\tau_i})\right\},$$

where $\widetilde{W}_{\tau_i} = (Z_{\tau_i}/U)^{1/\gamma}$ if $Z_{\tau_i} > UR^\gamma$ or $\widetilde{W}_{\tau_i} = (Z_{\tau_i}/L)^{1/\gamma}$ if $Z_{\tau_i} < UR^\gamma$.

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- $\frac{d\widetilde{W}_t}{\widetilde{W}_t} = 0$ if $Z_t \in [LR^\gamma, UR^\gamma]$.

Calibration Strategy

- Informed by our framework:
  - Endogenous effort $\Longrightarrow$ endogenous relationship between wage and output;
  - Link with the empirical literature on **"rent sharing elasticities"** (Card et al., 2018);

$$\varepsilon_{W,Z} = \frac{1}{\gamma}, \quad \varepsilon_{W,Y} = \frac{1}{\gamma + \varepsilon_{f,W}}, \quad \varepsilon_{W,\Pi} = \frac{1 - \frac{W_t}{Y_t}}{\gamma + \varepsilon_{f,W} - \frac{W_t}{Y_t}}$$

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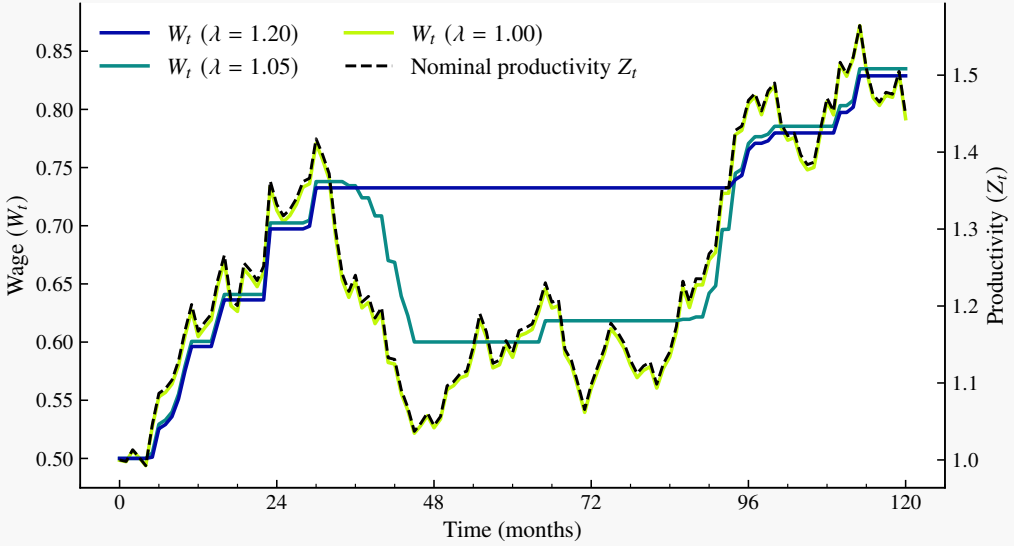
$$\varepsilon_{W,Z} = \frac{1}{\gamma}, \quad \varepsilon_{W,Y} = \frac{1}{\gamma + \varepsilon_{f,W}}, \quad \varepsilon_{W,\Pi} = \frac{1 - \frac{W_t}{Y_t}}{\gamma + \varepsilon_{f,W} - \frac{W_t}{Y_t}}$$

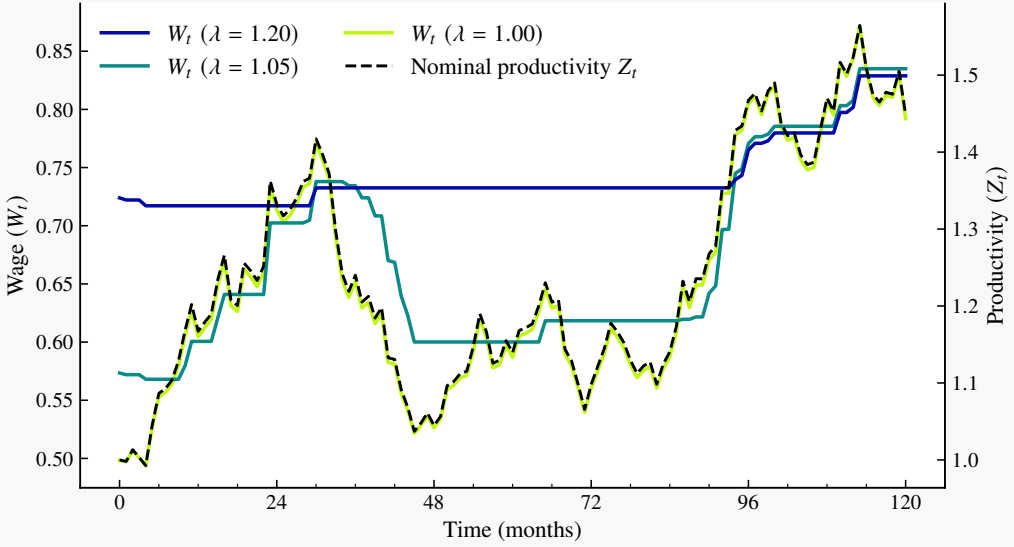
▶ Joint calibration to match empirical regularities as in Grigsby et al. (2021):

- Effort curvature + Effort sensitivity to wages $\rightarrow \varepsilon_{W,\Pi}$ (8%) and $\mathbb{E}\left[\frac{W_t}{Y_t}\right]$ (60%);
- Effort asymmetry $\rightarrow$ spike at zero/ $\mathbb{P}$(no wage change) (33%);
- GBM drift and volatility $\rightarrow$ median (2.4%) and std (6.3%) of Δ wage distribution.

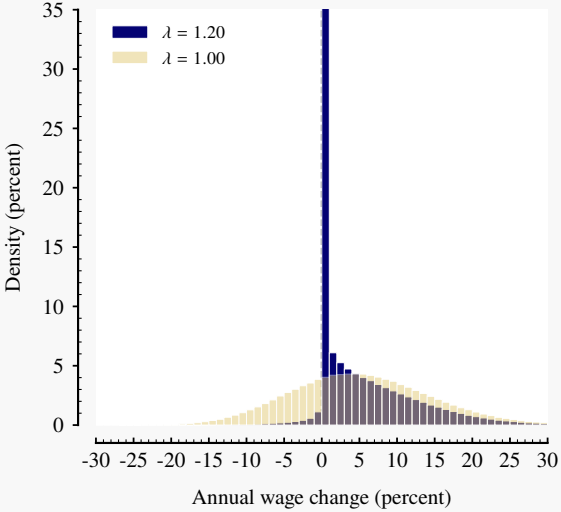
Wage dynamics (initial $R_0 = 0.5$)

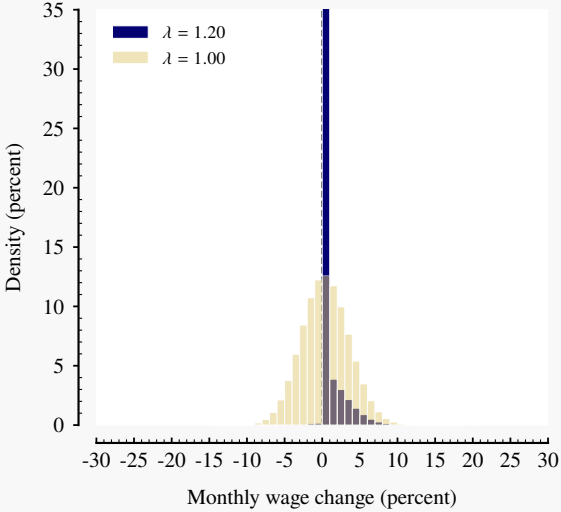


Wage dynamics (initial $R_0 = 1.0$)



Distribution of wage changes (calibrated vs counterfactual)





Baseline model calibration summary

Targeted Moments (annual frequency)	Model	Data	Data Source
Probability of no wage change (%)	33.2	33.2	Grigsby et al. (2021)
Median unconditional wage change (%)	2.4	2.4	Grigsby et al. (2021)
Std. dev. of unconditional wage change (%)	6.3	6.3	Grigsby et al. (2021)
Average rent-sharing elasticity (%)	8.0	8.0	See evidence discussed
Average labor share of income (%)	60	60	Standard
Calibrated Parameters	**Value**		**Comment**
Relative sensitivity of effort to wage cuts λ	1.20		See discussion in text
Real productivity drift μ (annual)	0.02		See discussion in text
Real productivity volatility σ (annual)	0.07		See discussion in text
Effort function curvature parameter γ	0.80		See discussion in text
Sensitivity of effort to wage changes η	17.3		See discussion in text
Assigned Parameters	**Value**		**Comment**
Time discount rate ρ (annual)	0.04		Standard
Exogenous job separation rate δ (annual)	0.24		24% annual separation rate
Inflation rate π (annual)	0.02		2% annual inflation rate
Poisson arrival rate ν	1.00		Expected arrival once per month

Baseline model calibration; additional statistics

Moments (monthly frequency)	Model
Mean unconditional wage change (%)	0.4
Median unconditional wage change (%)	0.0
Std. dev. of unconditional wage change (%)	1.3
Skewness of unconditional wage change	3.5
Kurtosis of unconditional wage change	17.9
Mean conditional wage change (%)	2.4
Median conditional wage change (%)	2.0
Std. dev. of conditional wage change (%)	2.1
Skewness of conditional wage change	0.7
Kurtosis of conditional wage change	4.7
Mean wage change conditional on positive (%)	2.5
Median wage change conditional on positive (%)	2.1
Std. dev. of wage change conditional on positive (%)	1.9
Skewness of wage change conditional on positive	1.1
Kurtosis of wage change conditional on positive	4.3
Mean wage change conditional on negative (%)	−2.1
Median wage change conditional on negative (%)	−1.7
Std. dev. of wage change conditional on negative (%)	1.6
Skewness of wage change conditional on negative	−1.1
Kurtosis of wage change conditional on negative	4.2

Conclusion & Future Directions

Going forward

- So far:
  - Analytical and closed-form solutions of prominent model of path-dependent wage rigidity + optimality and uniqueness;
  - Partial equilibrium simulation is promising for wage dynamics.

Going forward

- So far:
  - Analytical and closed-form solutions of prominent model of path-dependent wage rigidity + optimality and uniqueness;
  - Partial equilibrium simulation is promising for wage dynamics.
- Next steps:
  - Develop the GE model in the spirit of a DMP model with heterogeneous agents;
  - Incorporate endogenous separations, worker participation constraint, and characterize the equilibrium;
  - Conduct novel analysis of aggregate and distributional dynamics.

⊙ **Thank you for your time and attention** ⊙

Appendix / Additional Slides

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