Looking For Risk: Volatility Bounds in Macro

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Measuring Equity Premia

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  * VIX is a volatility measure, but often used to proxy for (global) "risk"

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3. The calibrated model-implied ERP outperforms the SVIX OOS at 3m horizons (4.84% vs 3.5 %).
4. Findings robust to other specifications:
   * e.g. non-lognormal environment (disasters), replicate Farhi-Gourio (2018) and show rise in ERP is not accompanied by significant rise in model-implied *SVIX*

Related Literature (Non Exhaustive)

- **Asset Pricing Bounds:** Epstein & Zin (1989), Hansen and Jagannathan (1991), Constantinides & Duffie (1996), Alvarez & Jermann (2004), Campbell (2008), Campbell and Thompson (2010), Van Binsbergen & Koijen (2010), Drechsler and Yaron (2011), Drechsler (2013), Bekaert & Hoerova (2014) Martin (2017), Chabi-Yo and Loudis (2020), Back, Crotty Kazempour (2022)
- **Macro:** Baker et al. (2014), Caballero, Farhi & Gourinchas (2017), Farhi & Gourio (2018), Kozlowski, Veldkamp & Venkateswaran (2019, 2020), Atkeson, Heathcote & Perri (2024), Greenwald, Lettau & Ludvigson (2025), Corsetti, Lloyd, Marin & Ostry (2026), Caramp & Singh (2026).

Asset Pricing Bounds Redux 1/2

Absent arbitrage, returns are determined by the Euler equations:

$$\mathbb{E}_t(M_{t+1}R_{t+1}) = 1, \quad \mathbb{E}_t(M_{t+1})\,R_{f,t} = 1$$

$M_{t+1} \geq 0$ SDF, R_{t+1} **risky return**, $R_{f,t}$ risk-free rate.

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The lower bound for ERP can be constructed as follows:

$$\mathbb{E}_t(R_{t+1}) - R_{f,t} = \frac{1}{R_{f,t}}\,Var_t^*(R_{t+1})\underbrace{-Cov_t(M_{t+1}R_{t+1},\,R_{t+1})}_{NCC}$$

$$\geq \underbrace{\frac{1}{R_{f,t}}\,Var_t^*(R_{t+1})}$$

$$\equiv R_{f,t}\,SVIX_{t,t+1}^2 \text{ [Martin (2017)]}$$

▸ How is the SVIX measured?

Asset Pricing Bounds Redux 2/2

ERP can thus be nested as follows:

$$\underbrace{\ln\left(Var_t^*\left(\frac{R_{t+1}}{R_{f,t}}\right)+1\right)}_{\mathcal{L}_t^b \text{ [Martin (2017)]}} \leq \ln\left(\frac{\mathbb{E}_t(R_{t+1})}{R_{f,t}}\right)$$

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$\mathcal{L}^b$: **example.** log utility consuming the aggregate dividend ($C_{t+1} = D_{t+1}$): $M_{t,t+1} = \beta\, C_t/C_{t+1}$ and $R_{t+1} = \frac{1}{\beta}\, D_{t+1}/D_t$, so $M_{t+1}R_{t+1} = 1 \;\Rightarrow\; Cov_t(MR, R) = 0$ and $ERP = \mathcal{L}^b$ [Lucas (1978)].

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How do these bounds vary in more general settings?

Model Setup

Environment. Discrete time $t \in \{0, 1, \dots\}$, representative shareholder with power utility, a **time-varying** price of risk x_t [Brennan, Wang & Xia (2004); Lettau & Wachter (2007, 2011); GLL] and discount factor β_t. SDF:

$$M_{t+1} = \beta_t \left(\frac{C_{t+1}}{C_t} \right)^{-x_t}$$

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Technology, shares & payouts. CRS production $Y_t = A_t K_t^{\alpha} L_t^{1-\alpha}$; a constant fraction ω of output is invested [GLL; Jermann (1998)]. The shareholder consumes the aggregate dividend:

$$C_t = D_t = E_t - \omega Y_t, \qquad E_t = S_t Y_t \implies :$$
$$\Delta c_{t+1} \approx \underbrace{\frac{\bar{S}}{\bar{S} - \omega}}_{\ell > 1} \Delta s_{t+1} + \Delta y_{t+1}.$$

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Processes: $z_t \equiv \left(s_t,\ x_t,\ \delta_t,\ g_t,\ \sigma^2_{x,t} \right)'$, normally distributed

Analytical Gap

Simple algebra yields:

$$ERP_t = x_t \operatorname{cov}_t(\Delta c_{t+1}, r_{t+1}),$$
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Gap can be expressed as a multiple of the total quantity of risk:

$$ERP_t - \mathcal{L}_t^b = \operatorname{cov}_t(\Delta c_{t+1}, r_{t+1})\left(x_t - \underbrace{\frac{\operatorname{Var}_t(r_{t+1})}{\operatorname{cov}_t(\Delta c_{t+1}, r_{t+1})}}_{x_t^*}\right),$$

where x_t^* is a "baseline" price of risk.

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* Only when $x_t = x_t^*$ is the return variance a sufficient statistic for priced risk – the Lucas (1978) economy with log utility $x_t = 1$

Gap Drivers

Based on a second-order SDF perturbation and a Campbell–Shiller (1988) approx.

$$m_{t+1} \simeq -\delta_t - \mu_t - x_t\,\Delta c_{t+1} + \bar{x}\,\ell(\ell-1)\big(\mathbb{E}_t[s_{t+1}] - \bar{s}\big)\Delta s_{t+1},$$
$$r_{t+1} = \kappa_0 + \kappa_1\, pc_{t+1} - pc_t + \Delta c_{t+1}, \qquad pc_t = A_0 + A'\tilde{z}_t,$$

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The implied gap is given as:

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with the return variance $\beta'\Sigma_t\beta$ decomposing as:

$$\beta'\Sigma_t\beta = b_c'\Sigma_t\beta + \underbrace{\kappa_1 A_{cf}'\Sigma_{cf}\big(b_c + \kappa_1 A_{cf}\big)}_{\text{priced-shock revaluation}} + \underbrace{\kappa_1^2\big(A_x'\Sigma_{x,t}A_x + A_\delta'\Sigma_\delta A_\delta + B_x'\Sigma_\sigma B_x\big)}_{\text{unpriced shocks}}.$$

Quantification and Measurement (GLL)

We relate the state vector to the observable series:

$$\mathcal{Y}_t = \left(ey_t,\ r_{f,t},\ \bar{r}^{40}_{f,t},\ \Delta y_t,\ \bar{g}^{40}_t,\ py_t,\ \mathcal{L}^b_t\right)'$$

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\bar{g}_t^{40} &= \bar{g} + \tfrac{1}{40}\,1'(I - \Phi_g)^{-1}(I - \Phi_g^{40})\,\tilde{g}_t + \nu_g, \\
py_t &= \overline{py} + (A_s + \ell 1)'\tilde{s}_t + A_x'\tilde{x}_t + A_\delta'\tilde{\delta}_t + A_g'\tilde{g}_t, \\
\mathcal{L}_t^b &\equiv \ln(1 + SVIX_t^2) = \underbrace{\left(\Psi + \sigma_a^2\right)\left(\bar{x} + 1'\tilde{x}_t\right) - \Psi\Gamma'\tilde{s}_t}_{\boldsymbol{ERP_t}},
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\mathcal{L}_t^b &\equiv \ln(1 + SVIX_t^2) = \mathrm{Var}R_0 + \mathrm{Var}R_v'\,\tilde{\sigma}_{x,t}^2,
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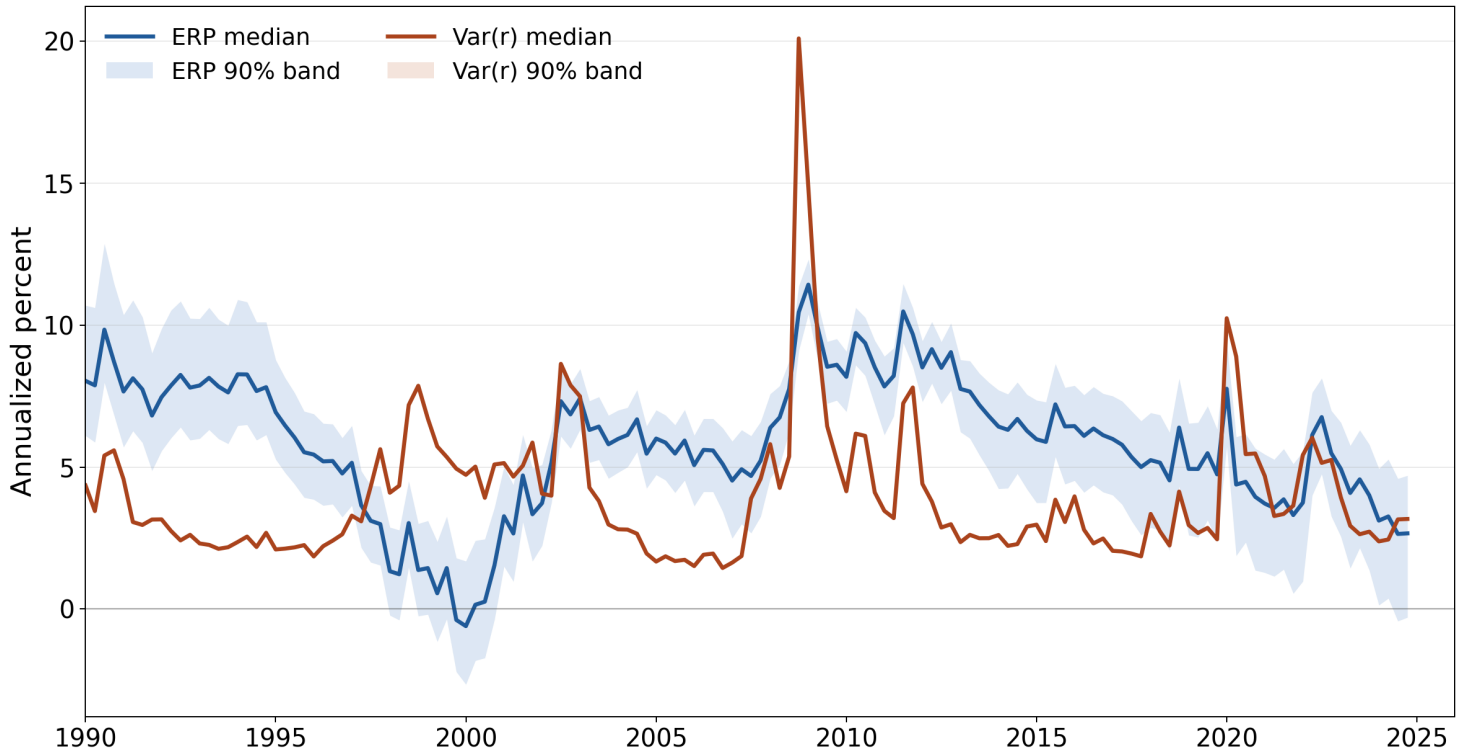
Model-implied ERP vs. the SVIX bound



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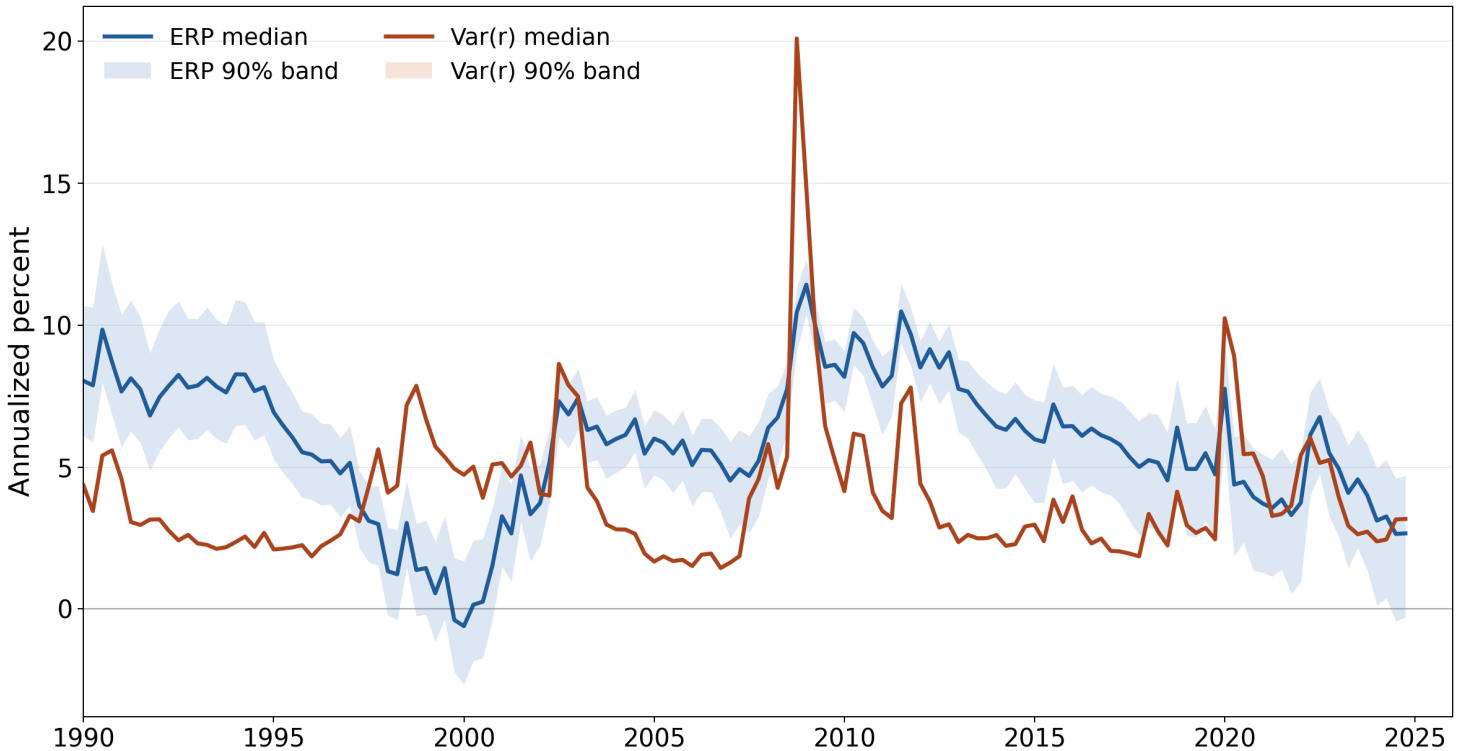
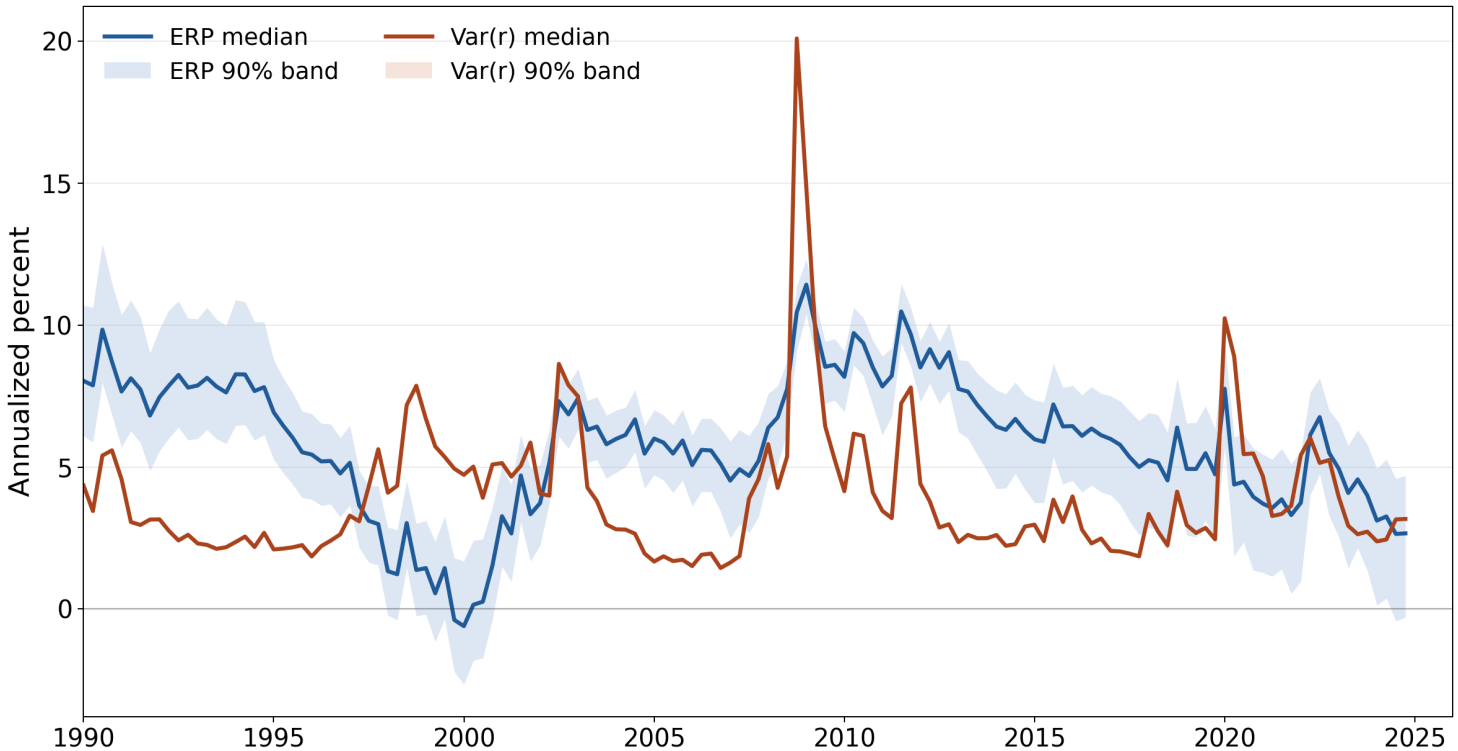


* On average ERP exceeds the SVIX bound (5.48% vs. 3.76%)

Model-implied ERP vs. the SVIX bound



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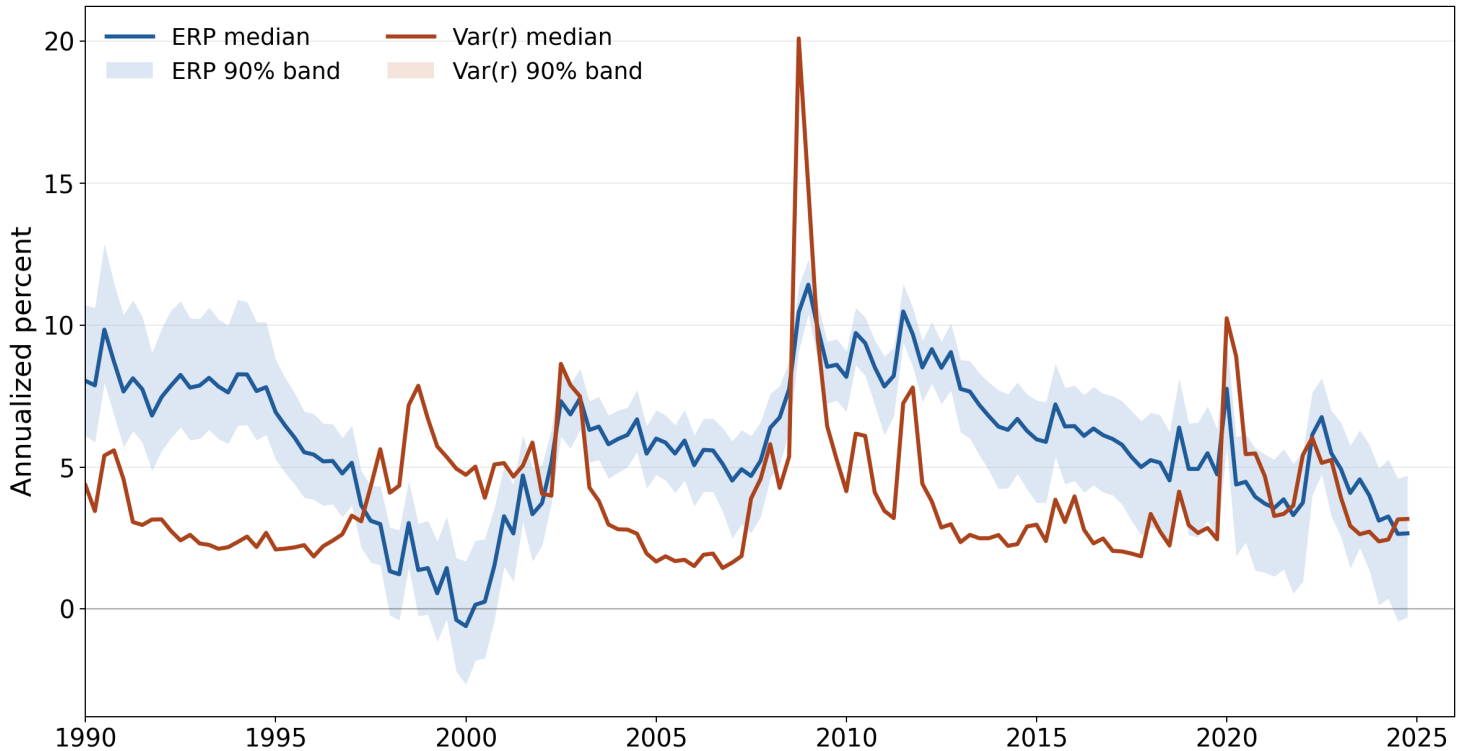

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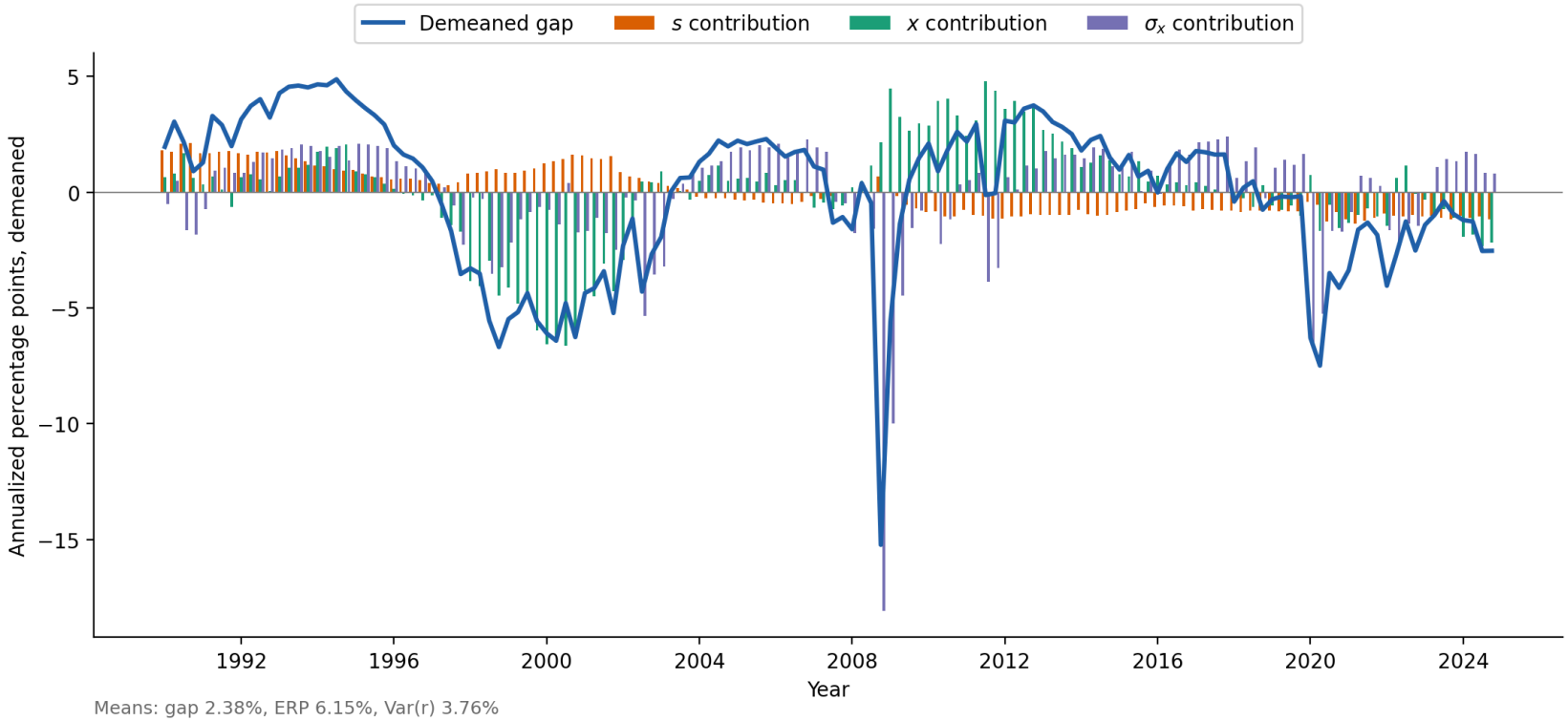
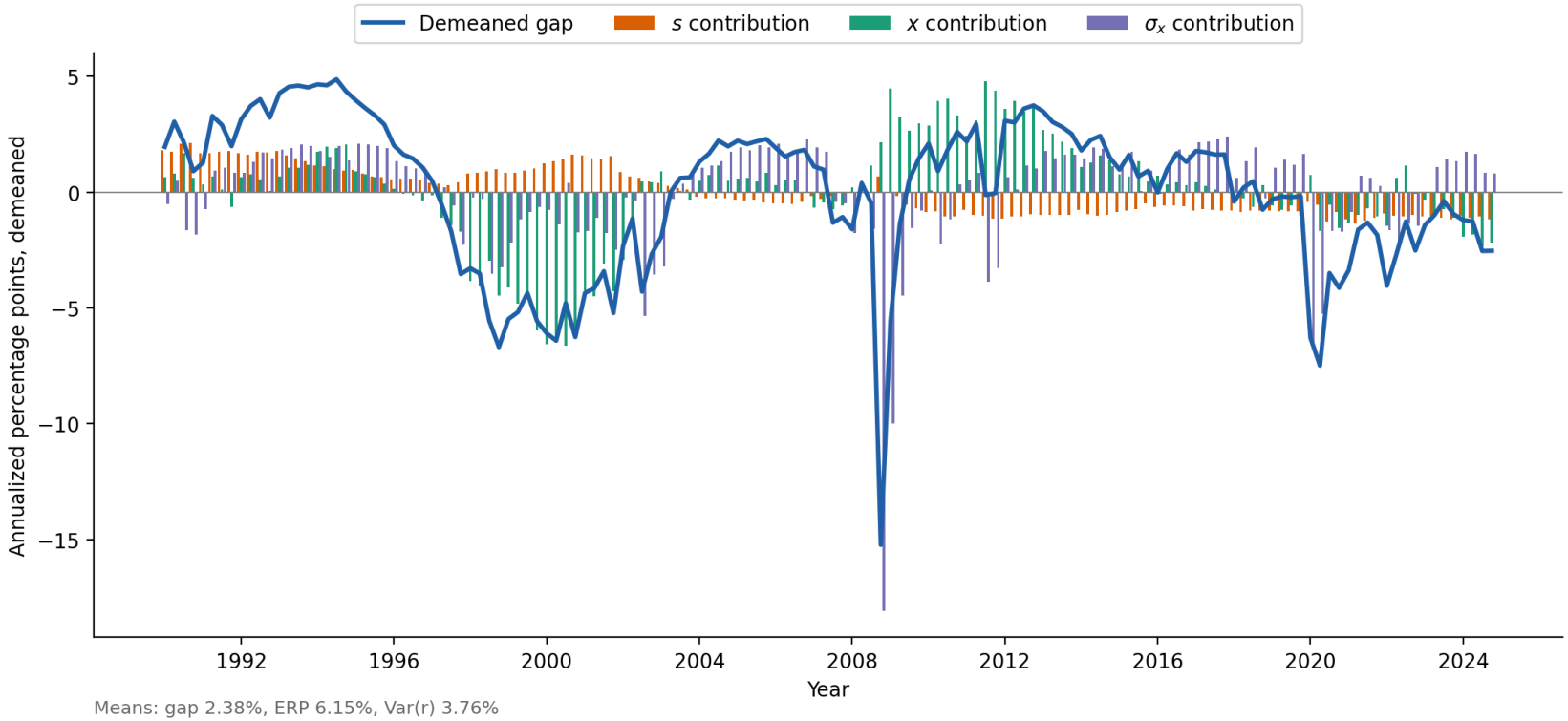
* ERP falls below SVIX in crises (dot-com, GFC, COVID) as $\mathcal{L}^b$ spikes
* Record valuations *py* at end of sample & *s* flat imply *x* low $\Rightarrow$ ERP $\downarrow$

Decomposing the Gap over Time



Means: gap 2.38%, ERP 6.15%, Var(r) 3.76%

Decomposing the Gap over Time



* **1990s:** *py* and *s* both low $\Rightarrow$ *x* above mean; options calm $\Rightarrow$ gap wide

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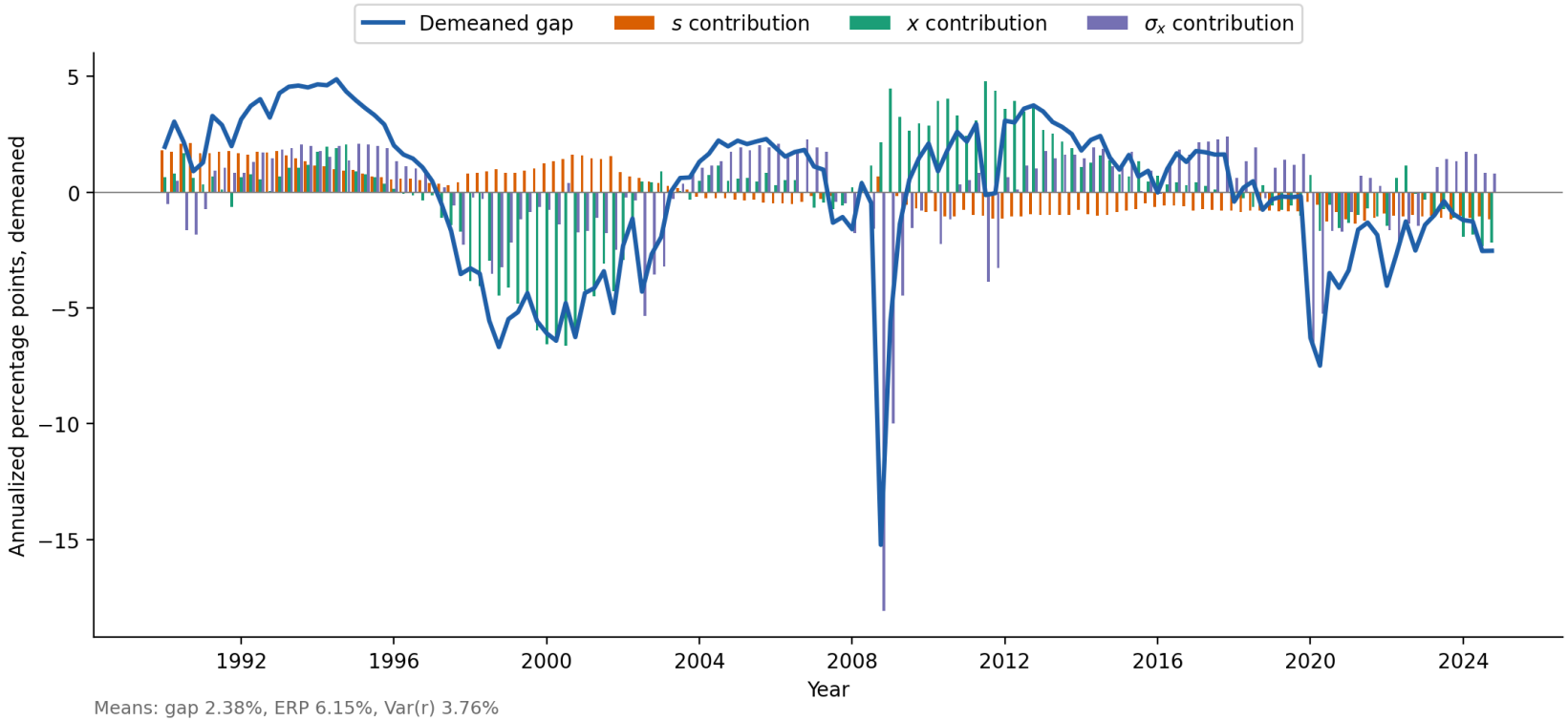

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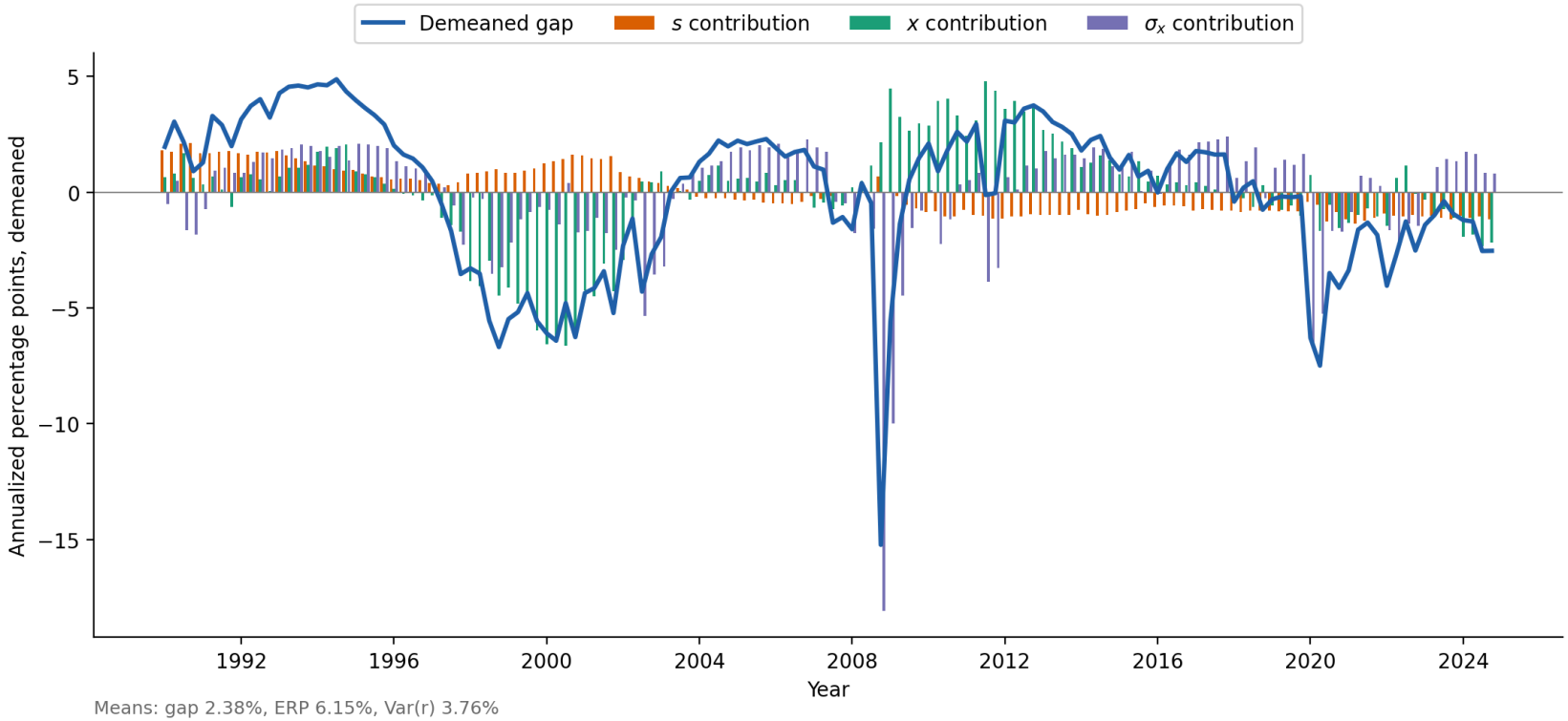

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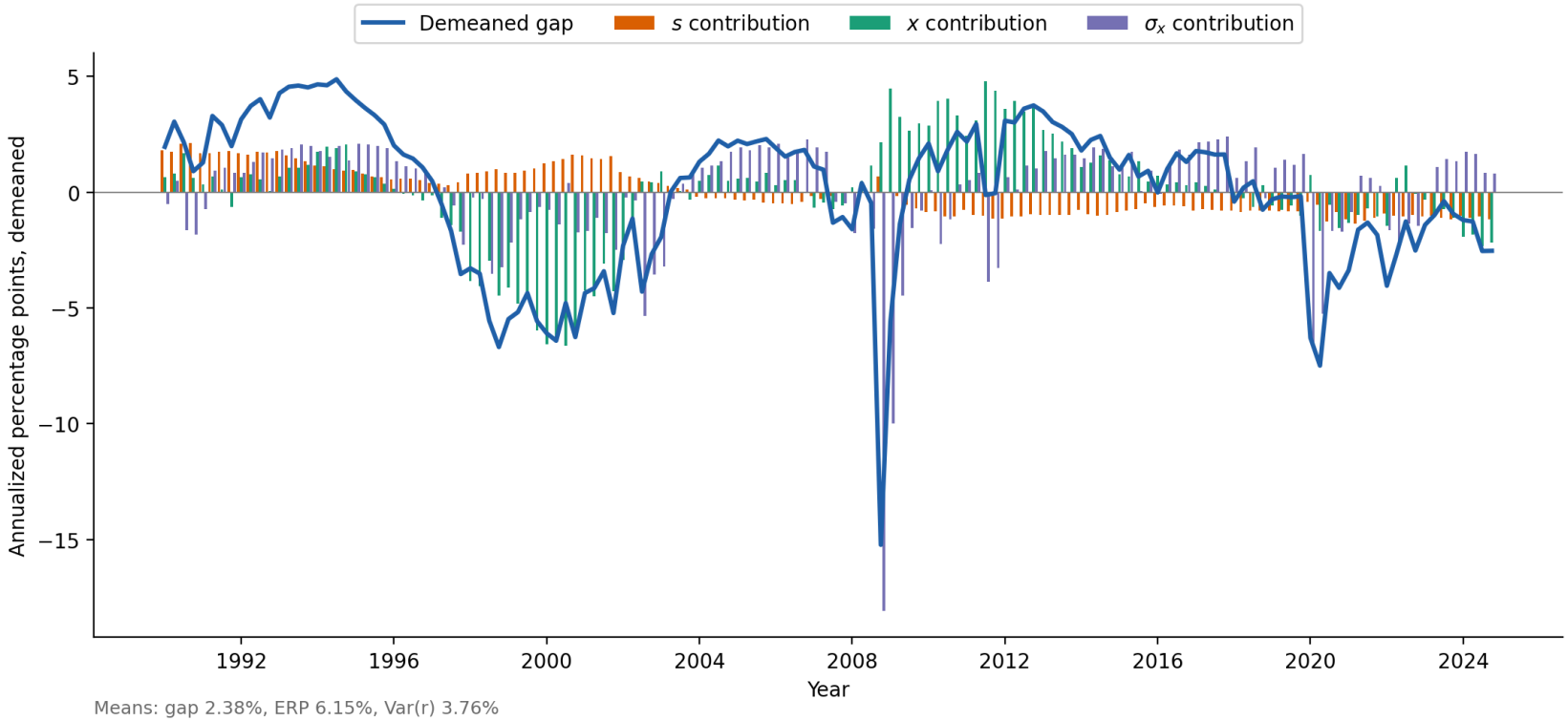

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$$R^2_{OS} = 1 - \sum_t \varepsilon_t^2 / \sum_t \nu_t^2$$

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* Fitted ERP on $\log(P/Y)$ and $s = \log(E/Y)$:

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* Extended [Abel, 1999] i.i.d economy & short-term equity:

  $ERP_t - \mathcal{L}_t^b \sim$ correl. of valuations & returns, risk aversion, 1/leverage

  - extensions to long-term equity & non-market income

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* The calibrated model-implied gap has OOS forecasting power on realized returns ($> 5\%$ quarterly)

Lesson for theory and policy: ERP & price of risk can evolve independently of volatility indices

Measuring $SVIX^2$ with option contracts [Martin (2017)]

SVIX constructed using portfolio of out-of-the-money options weighted *equally* across strikes:

$$SVIX^2_{t,T} = \frac{2}{(T-t)\,R_{f,t}\,S_t^2}\left[\int_0^F \mathrm{put}_t(K)\,dK + \int_F^\infty \mathrm{call}_t(K)\,dK\right]$$

* F: forward price, S : spot price
* In practice, construct integral using sum of puts and calls

back