The Empirical Distribution of Marshall's Second Law of Demand

Nathan G. Hattersley[1] Eugenio J. Miravete[2]

[1]University of Texas at Austin & CEPR

[2]University of Texas at Austin

24th CRETE Conference, Ios, July 2026

About Marshall's 2nd Law of Demand

- Let's refresh our memory...

About Marshall's 2nd Law of Demand

- Let's refresh our memory...

Alfred Marshall (1920): Principles of Economics, B.III; Ch.IV, §2

"The elasticity of demand is great for high prices, and great, or at least considerable, for medium prices; but it declines as the price falls; and gradually fades away if the fall goes so far that satiety level is reached."

About Marshall's 2nd Law of Demand

- Let's refresh our memory...

Alfred Marshall (1920): Principles of Economics, B.III; Ch.IV, §2

"The elasticity of demand is great for high prices, and great, or at least considerable, for medium prices; but it declines as the price falls; and gradually fades away if the fall goes so far that satiety level is reached."

- ... or with a more concise wording, $\varepsilon_p > 0$ where ε is the elasticity of the inverse demand function.

Questions

- **Q1:** How fast does demand become more elastic with price?
- **Q2:** Why do we care?
- **Q3:** How representative are the functional forms that we commonly assume for demand in our economic models?
- **Q4:** How can we answer the previous question if we estimate a particular functional form for demand?
- **Q5:** Can we characterize the distribution of pass-through or the distribution of frictions that enables an effective monetary policy?

Agenda

Topics to cover:

1. Connecting Demand Curvature, Superelasticity and Marshall's 2nd Law.
2. Invariant Demand Manifolds.
3. IRI Data.
4. Nonparametric Demand Estimation.
5. Empirical Distribution of Demand Shapes.
6. Correlation Between Demand Elasticity and Curvature.
7. Distribution of Pass-Through.
8. Distribution of Superelasticity.

Demand Subconvexity

- Elasticity:

$$\varepsilon(x) \equiv -\frac{p(x)}{x \cdot p'(x)} = -\frac{p \cdot x'(p)}{x(p)} \,,$$

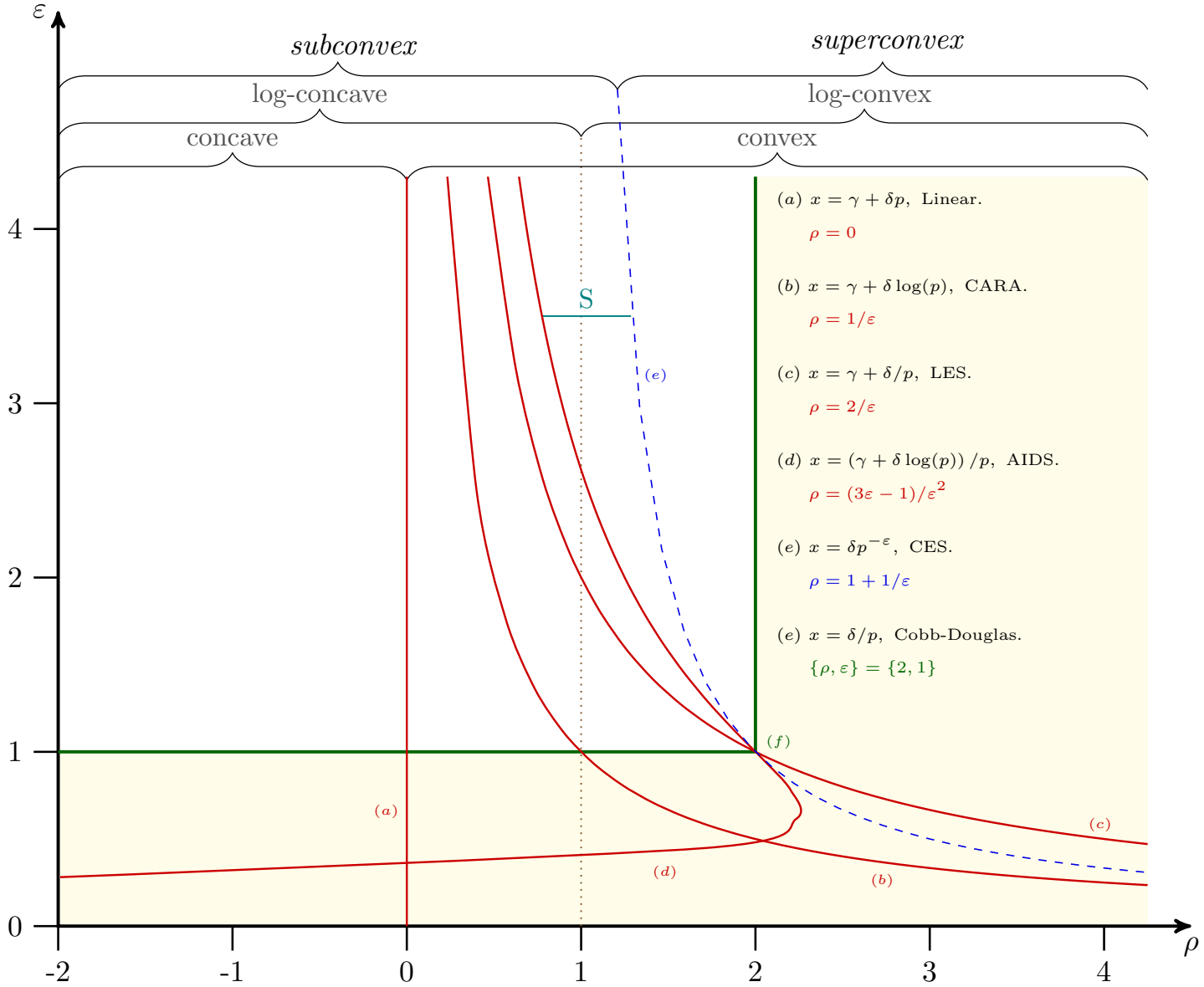
- Curvature:

$$\rho(x) \equiv= -\frac{x \cdot p''(x)}{p'(x)} = -\frac{x(p) \cdot x''(p)}{[x'(p)]^2} \,,$$

- Marshall's 2nd Law and Demand Subconvexity:

$$\begin{aligned} \varepsilon_p &= -\frac{x' \cdot \varepsilon}{x}\left(1 + \frac{1}{\varepsilon} - \rho\right) \,, \\ &= -\frac{x'}{x} \cdot (\varepsilon + 1 - \varepsilon\rho) \,, \\ &= -\frac{x'}{x} \cdot S > 0 \,. \end{aligned}$$

Figure 1: Invariant Demand Manifold



Data

Table 1: Descriptive Statistics

	Beer	C.Cereal	Carb.Bevs.	Coffee	Fzn.Dinner	Fzn.Pizza	H.Cleaner	S.Snacks	Soup	Yogurt
Price ($)										
Average Price	9.51	3.53	2.55	6.39	3.28	4.10	3.44	2.79	1.91	1.62
Std. Dev. Price	4.62	1.04	1.63	2.71	2.07	1.95	1.43	1.17	0.91	1.38
Maximum Price	29.99	23.48	136.86	18.16	18.99	12.49	8.59	11.98	9.08	12.98
Minimum Price	0.39	0.02	0.01	0.05	0.01	0.01	0.02	0.01	0.01	0.01
Supermarket Chains										
Avg. Number Stores	37.63	25.89	25.87	25.89	25.85	25.89	25.92	25.89	25.87	25.85
Avg. Number UPCs	173.50	198.79	201.74	197.77	199.09	186.43	191.6	201.21	201.87	160.34
Avg. Weekly Sales (units)	14,336.13	35,387.87	87,785.39	9,472.13	38,735.87	15,559.43	6,468.78	38,822.46	58,744.50	72,762.75
Avg. Weekly Sales (dollars)	133,745.64	106,185.13	188,511.02	53,512.08	94,513.01	45,947.98	19,094.88	94,982.34	73,173.88	69,776.44
UPC-Chain Price Dispersion										
Avg. Coefficient of Variation	0.08	0.14	0.15	0.12	0.14	0.14	0.11	0.11	0.15	0.12
Max. Coefficient of Variation	0.53	0.55	1.19	0.70	0.41	0.41	0.64	0.47	0.95	0.46
Min. Coefficient of Variation	0.00	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Overall Sample										
Total Number of Chains	52	53	53	53	53	53	53	53	53	53
Total Number of Supermarkets	989	1021	1021	1020	1020	1020	1020	1021	1021	1020
Total Number of UPCs	1,747	1,821	3,199	2,364	1,772	959	986	2,868	2,153	1,598
Total Sales (millions of units)	152.13	384.14	951.69	103.02	420.29	168.21	70.09	422.46	637.21	789.41
Total Sales (millions dollars)	1,418.37	1,152.51	2,041.24	581.82	1,025.15	495.46	207.08	1,034.15	794.67	757.49
Total Number of Observations (millions)	23.49	32.74	43.40	20.80	50.16	14.75	15.53	33.97	51.86	29.43

Notes: Means of the winsorized sample. The coefficient of variation is the ratio of the within chain standard deviation of prices divided by the mean chain price.

- IRI 2008-2011; 10 categories; 50+ chains; 22,235 UPCs; 50 large cities. We will focus on "cold cereal."

NP Estimates

- DellaVigna-Gentzkow (2019):

$$\log\left[x(p_{sjt})\right] = \varepsilon_{jt}\log\left[p_{sjt}\right] + \alpha_{sjy} + \gamma_{sjw} + \epsilon_{sjt}\,,$$

- Although perfectly correct for their goal of evaluating foregone profits of 3DPD, the adopted CES demand restricts the value of the estimated curvature: $\rho = 1 + 1/\varepsilon$. Their approach also requires estimating a very large number of weekly fixed effects for store-product demands with a very limited sample size.

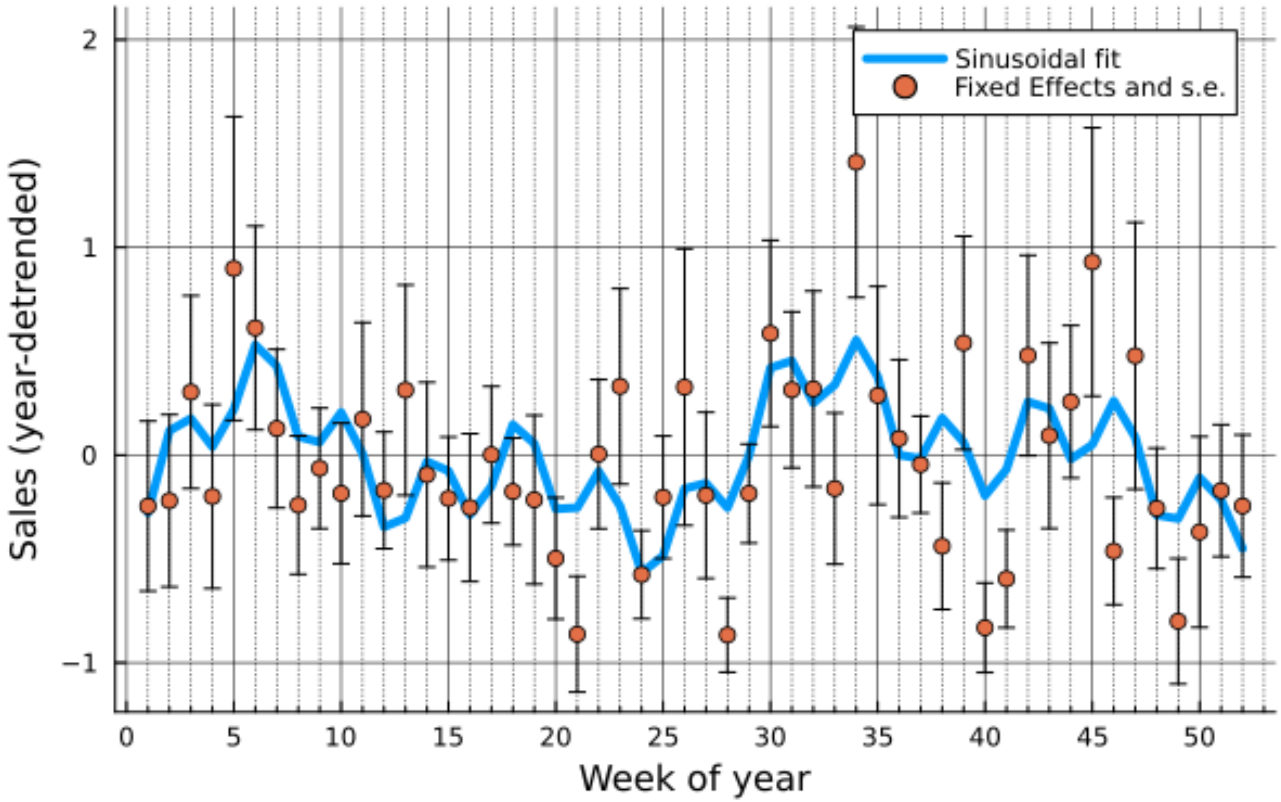
- Our approach:

$$x_{jt} = \varphi(p_{jt}) + g(t) + \epsilon_{jt}\,,$$

- Nonparametric specification with a functional form depending on a handful of parameters to deal with seasonality. Focus on chain-product demand estimation of supermarket chains present in more than one city to control for price endogeneity using the Hausman instruments.

Seasonality

Figure 2: Seasonality: Sinusoidal Regression vs. Weekly Fixed Effects

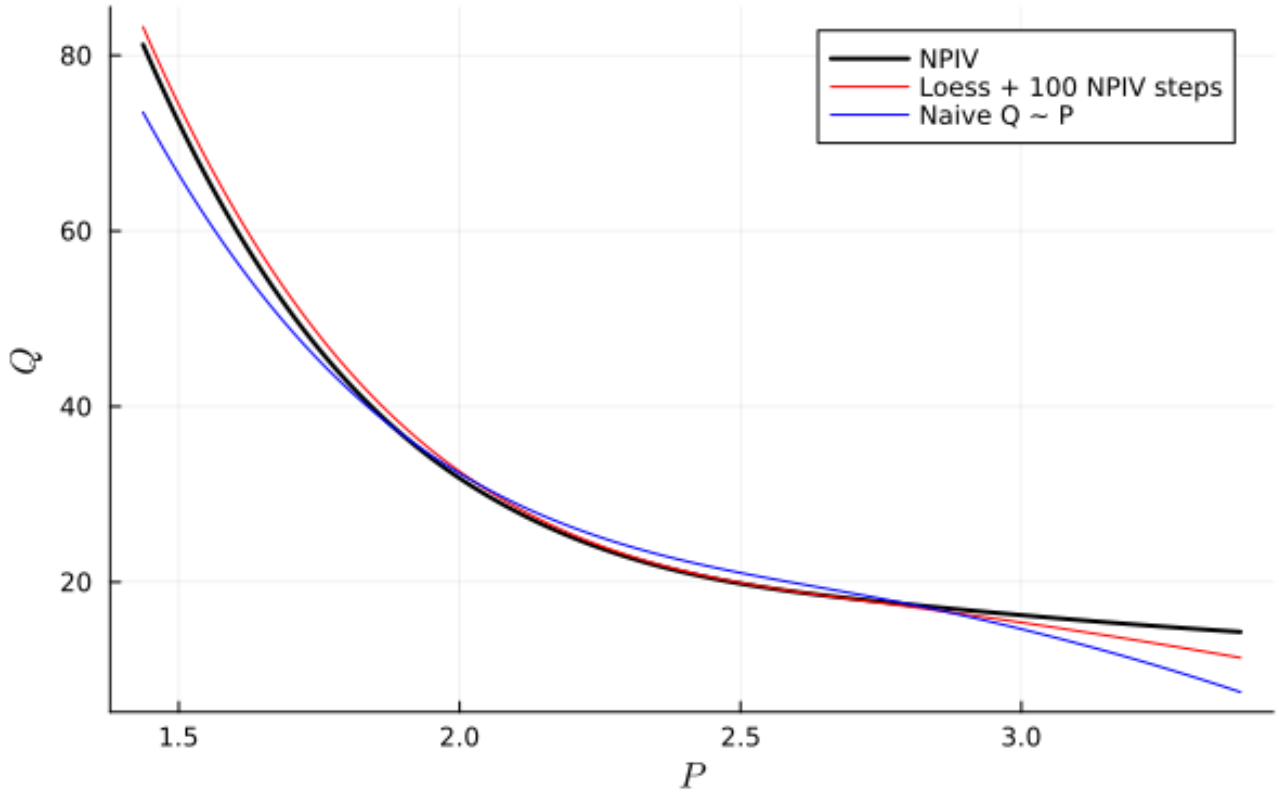


- Inverse Discrete Fourier Tansform (approx.):

$$g(t) = \alpha_{\text{year}(t)} + \sum_T \gamma_T \cos(2\pi \cdot \text{week}(t)/T) + \delta_T \sin(2\pi \cdot \text{week}(t)/T)\,, \quad T \in \{4, 13, 26, 52\}\,.$$

NPIV

Figure 3: Nonparametric Instrumental Regression



- Fredholm Integral Equation of the 1st kind: $\mathbb{E}[\tilde{\varphi}_{jt} - \varphi(p_{jt}) \mid z_{jt}] = 0$, (after Hausman instruments).
- Ill-posed inverse problem: $\hat{\varphi}^{(i+1)} = \hat{\varphi}^{(i)} + c \cdot \mathbb{E}\left[\mathbb{E}\left[\tilde{\varphi} - \hat{\varphi}^{(i)} \mid z\right] \,\middle|\, p\right] \quad c \in (0, 1)$.
- LOESS: $\mathbb{E}[Y \mid X]$, *locally estimated scaterplot smoothing*.

Derivative Smoothing

- Iterative NPIV generates a sequence $\{\hat{x}_{jt}, p_{jt}\}_{t=1}^{T}$ rather than a function. To interpolate and smooth higher-order derivatives (in addition to the demand function itself) we use a P-spline to estimate $\hat{\phi}$ using the $(T \times L)$ B-spline basis for p where the $(T \times 1)$ column vector $B_\ell(p_{jt})$ is the ℓth basis function.
- Derivatives:

$$\widehat{x'}(p_{jt}) = \sum_{\ell=1}^{L} B'_\ell(p_{jt}) \cdot \hat{\phi}_\ell \,, \qquad \widehat{x''}(p_{jt}) = \sum_{\ell=1}^{L} B''_\ell(p_{jt}) \cdot \hat{\phi}_\ell \,.$$

- Pointwise elasticity-curvature estimates:

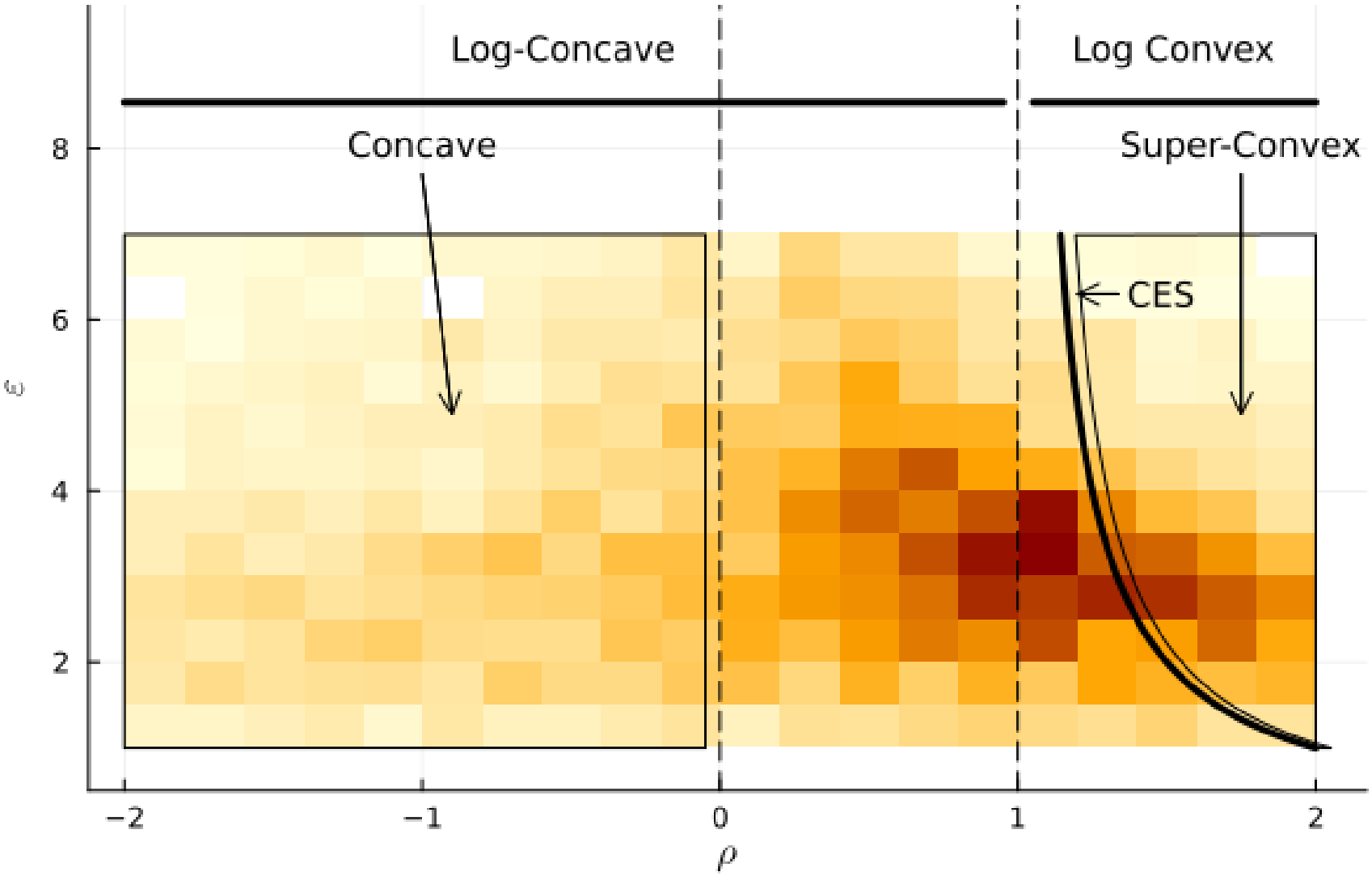
$$\hat{\varepsilon}_{jt} = -\widehat{x'}(p_{jt}) \cdot \frac{p_{jt}}{x_{jt}} \,. \qquad \hat{\rho}_{jt} = \frac{x_{jt} \cdot \widehat{x''}(p_{jt})}{\left(\widehat{x'}(p_{jt})\right)^2} \,.$$

- Chain-sales weighted product estimates:

$$\hat{\varepsilon}_j = \frac{\sum_{t=1}^{T} x_{jt} \cdot \hat{\varepsilon}_{jt}}{\sum_{t=1}^{T} x_{jt}} \,, \qquad \hat{\rho}_j = \frac{\sum_{t=1}^{T} x_{jt} \cdot \hat{\rho}_{jt}}{\sum_{t=1}^{T} x_{jt}} \,.$$

Results

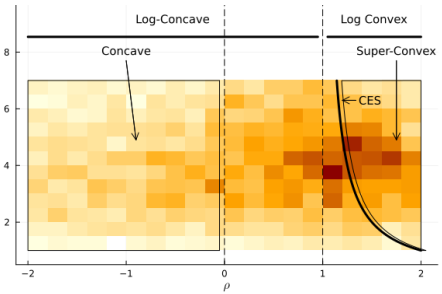
Figure 4: Distribution of Elasticity and Curvature (Cold Cereal)



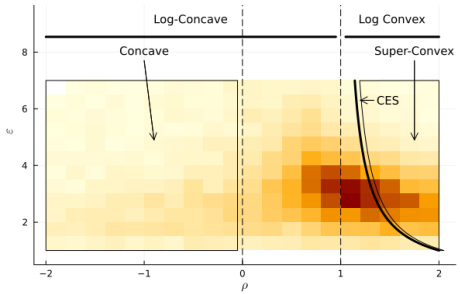
Results

Figure A.3: Distribution of Elasticity and Curvature Estimates

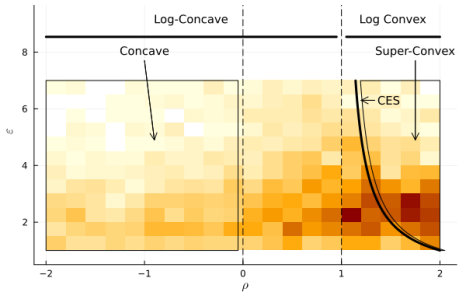
(a) Beer



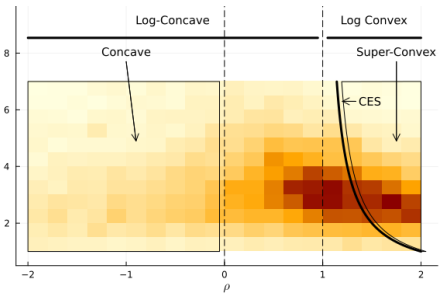
(b) Carbonated Beverages



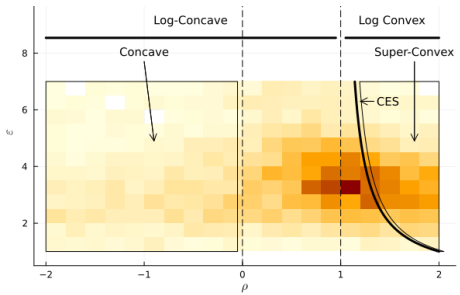
(c) Coffee



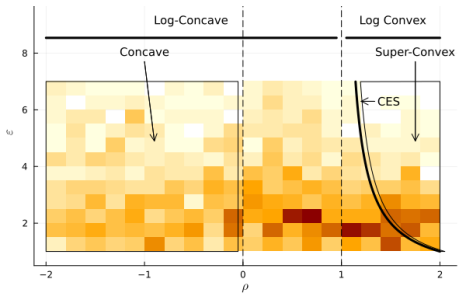
(d) Fzn.Dinner



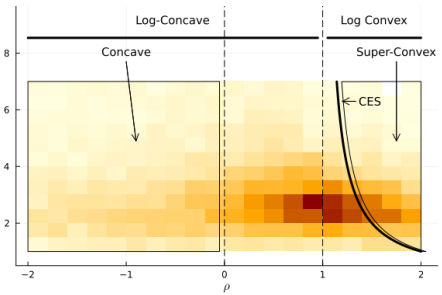
(e) Fzn.Pizza



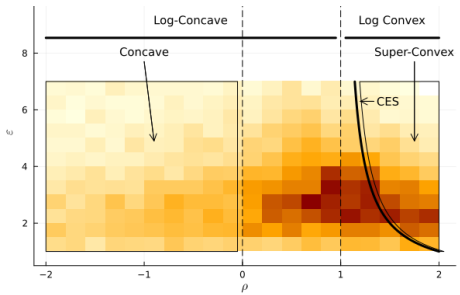
(f) H.Cleaner



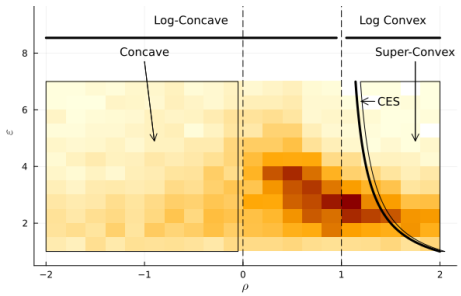
(g) S.Snacks



(h) Soup



(i) Yogurt



Results

Table 2: Empirical Distribution of Demand Shapes

	Concave	Log-concave		Log-convex	Convex	Subconvex	Superconvex
Demand Region	$\rho \leq 0$	$0 \leq \rho \leq 1$	$\rho \leq 1$	$1 \leq \rho \leq 1 + 1/\varepsilon$	$0 \leq \rho$	$\rho \leq 1 + 1/\varepsilon$	$1 + 1/\varepsilon \leq \rho$
Beer	31.24	31.88	63.12	11.73	43.61	74.84	25.16
Carbonated Beverages	19.45	37.54	56.98	19.54	57.08	76.52	23.48
Coffee	27.40	30.98	58.39	17.52	48.51	75.91	24.09
Cold Cereal	27.77	39.91	67.67	14.72	54.63	82.40	17.60
Frozen Dinners/Entrees	25.14	35.97	61.12	16.03	52.01	77.15	22.85
Frozen Pizza	21.88	35.79	57.67	16.63	52.42	74.30	25.70
Household Cleaner	41.96	31.01	72.97	12.81	43.82	85.78	14.22
Salty Snacks	29.93	37.10	67.04	16.46	53.57	83.50	16.50
Soup	29.85	35.44	65.29	15.27	50.71	80.56	19.44
Yogurt	31.13	42.17	73.30	14.16	56.32	87.46	12.54

Notes: Percentage of chain-product demand estimates by manifold region.

Table 3: Frequency of Particular Demand Specifications (Cold Cereal)

	Bandwidth	Linear	*CARA*	*LES*	*AIDS*	*CES*	Cobb-Douglas
Cold Cereal	0.10	5.14%	7.19%	9.05%	10.40%	7.61%	0.02%
$BW:$	0.05	1.0891	1.5007	1.8320	2.1639	1.4659	0.0000
$BW:$	0.10	1.0290	1.4374	1.8150	2.0890	1.5298	0.4842
$BW:$	0.20	1.0432	1.4010	1.8510	2.0310	1.5410	0.4842
$BW:$	0.50	1.1119	1.4273	1.7322	1.8549	1.5560	0.9878

Notes: The first line of each product category indicates the percentage of estimates within a 0.1 curvature bandwidth of the manifold of each demand family. The rest of each panel reports the density ratios of the empirical distribution of the estimated $(\hat{\varepsilon}, \hat{\rho})$ relative to a bi-variate uniformly distributed (ε, ρ).

$\text{corr}(\varepsilon, \rho)$

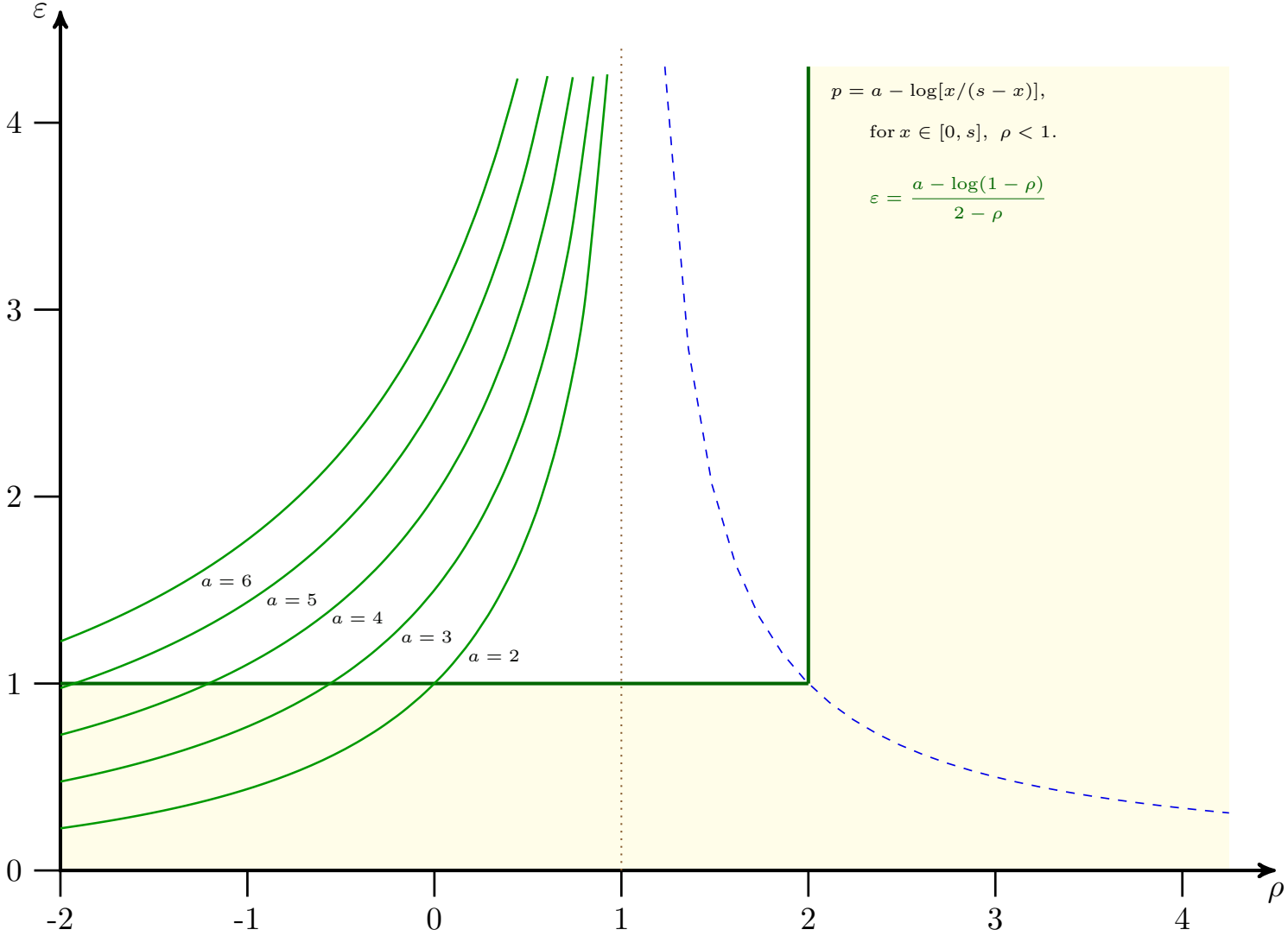
Table 4: Elasticity-Curvature Correlations

	Concave		Log-concave			Log-convex		Convex		Subconvex		Superconvex	
Demand Region	$\rho \leq 0$		$0 \leq \rho \leq 1$		$\rho \leq 1$	$1 \leq \rho \leq 1 + 1/\varepsilon$		$0 \leq \rho$		$\rho \leq 1 + 1/\varepsilon$		$1 + 1/\varepsilon \leq \rho$	
Beer	0.0301		0.0149		0.1071 (+)	-0.5116	(−)	-0.1741	(−)	0.0109		-0.2939	(−)
Carbonated Beverages	0.0833	(+)	-0.0343	(−)	0.1434 (+)	-0.4951	(−)	-0.2611	(−)	-0.0062		-0.3563	(−)
Coffee	0.1176	(+)	0.0552		0.1693 (+)	-0.4682	(−)	-0.1863	(−)	0.0382		-0.4283	(−)
Cold Cereal	0.1419	(+)	-0.0672	(−)	0.1038 (+)	-0.5074	(−)	-0.2698	(−)	-0.0311	(−)	-0.3846	(−)
Frozen Dinners/Entrees	0.0889	(+)	-0.0613	(−)	0.0702 (+)	-0.5016	(−)	-0.2581	(−)	-0.0578	(−)	-0.3678	(−)
Frozen Pizza	0.0866	(+)	-0.0404		0.1637 (+)	-0.4823	(−)	-0.2167	(−)	0.0496	(+)	-0.3150	(−)
Household Cleaner	-0.0672		-0.0817		-0.0202	-0.4226	(−)	-0.3180	(−)	-0.1518	(−)	-0.3554	(−)
Salty Snacks	0.0274		-0.0627	(−)	0.0025	-0.4715	(−)	-0.2451	(−)	-0.1076	(−)	-0.3760	(−)
Soup	0.0582	(+)	0.0284		0.1034 (+)	-0.4951	(−)	-0.2041	(−)	-0.0173		-0.4384	(−)
Yogurt	0.0961	(+)	-0.1774	(−)	0.0322 (+)	-0.4803	(−)	-0.3637	(−)	-0.1063	(−)	-0.3652	(−)

(+) and (−) indicate that the estimated correlation cannot rejected to be positive or negatively, respectively, based on the 95% confidence intervals Table A.2 in the Appendix reports the confidence intervals for each correlation estimate.

- Correlations are mostly negative except when we consider concave demands.
- This includes the log-concave region where logit is defined.
- Market concentration favors higher pass-through rates (not necessarily larger than 100%).

Results

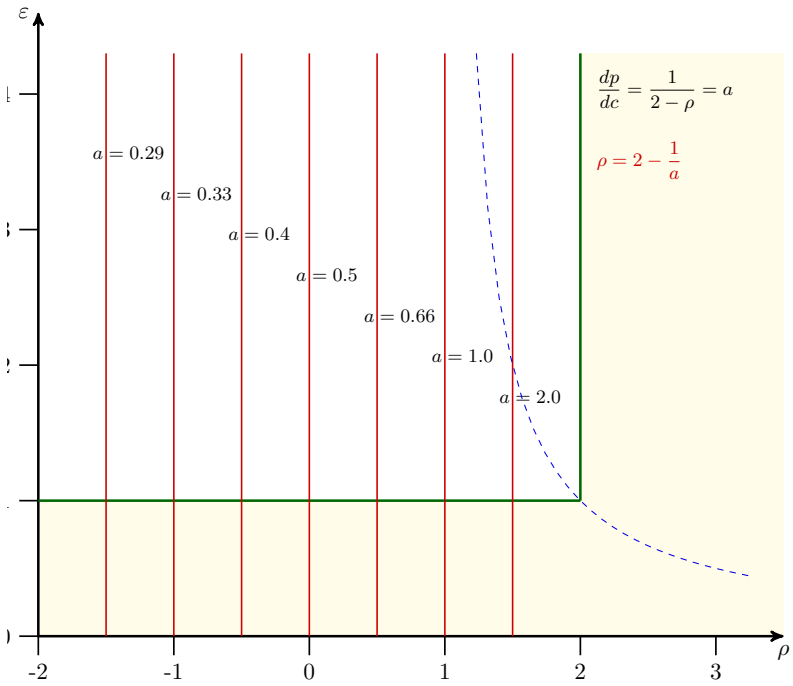


- Standard logit cannot accommodate this feature of the data unless we allow for price random coefficients.

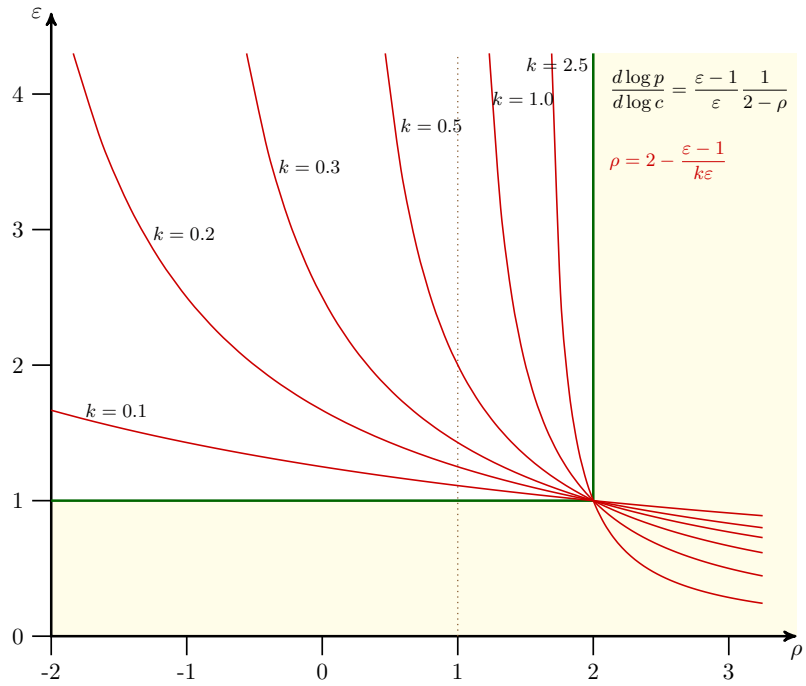
Pass-Through

Figure 6: Pass-Through Manifolds

(a) Absolute



(b) Proportional

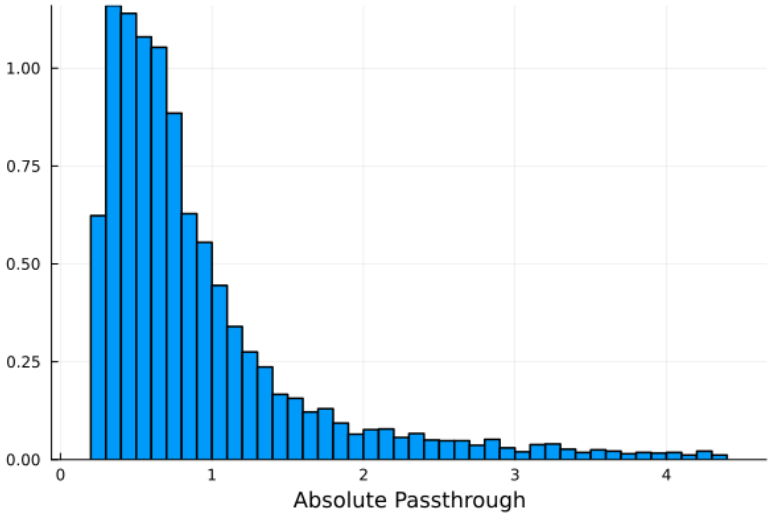


- Absolute pass-through is incomplete if demand is log-concave.
- Relative pass-through is incomplete if demand is subconvex (Marshall's).

Pass-Through

Figure 7: Distribution of Pass-Through Estimates (Cold Cereal)

(a) Absolute



(b) Proportional

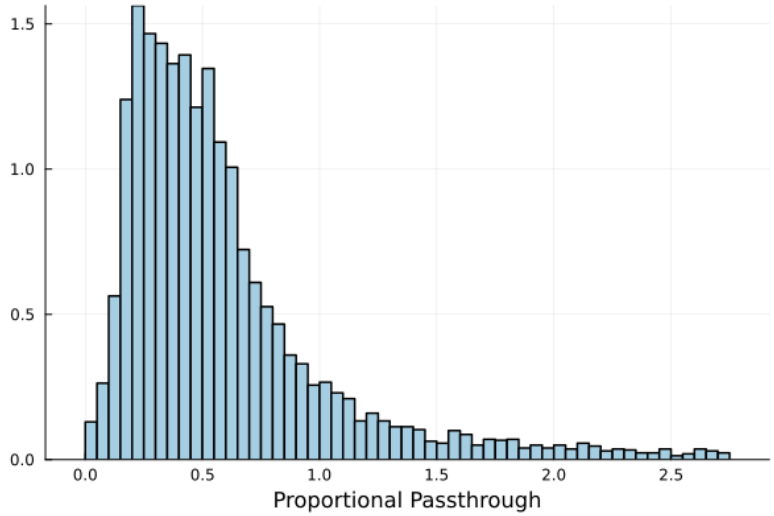


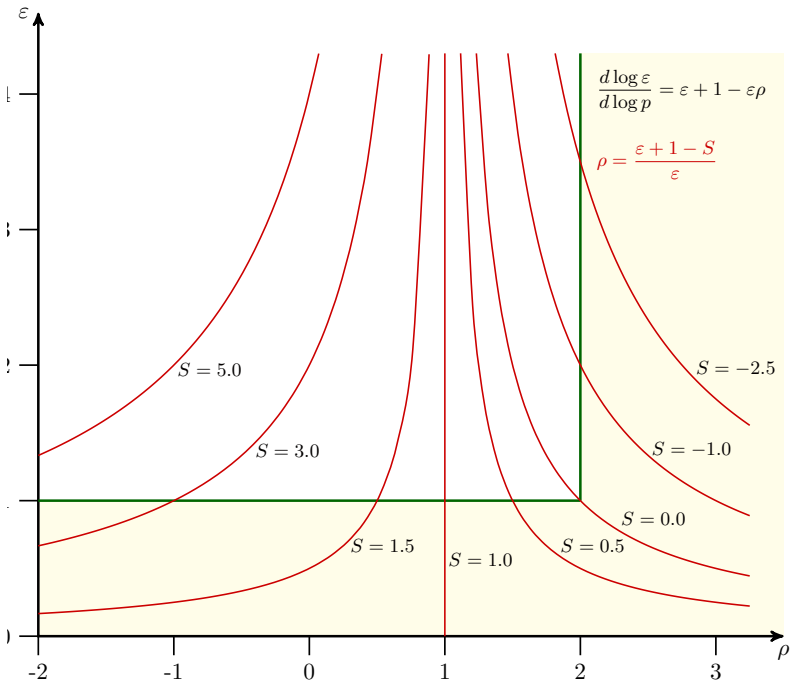
Table 5: Empirical Distribution of Pass-Through (Cold Cereal)

	Absolute Pass-Through			Proportional Pass-Through		
Elasticity Range	% Incomplete	Median	Mean	% Below 100%	Median	Mean
$[4, +)$	83.16	0.6339	1.0393	88.92	0.5120	0.8321
$[3, 4)$	64.53	0.7781	1.4209	80.94	0.5635	1.0262
$[2, 3)$	59.02	0.8419	1.9169	75.55	0.5387	1.2317
$[1, 2)$	63.96	0.7157	2.4910	84.17	0.3042	1.0738

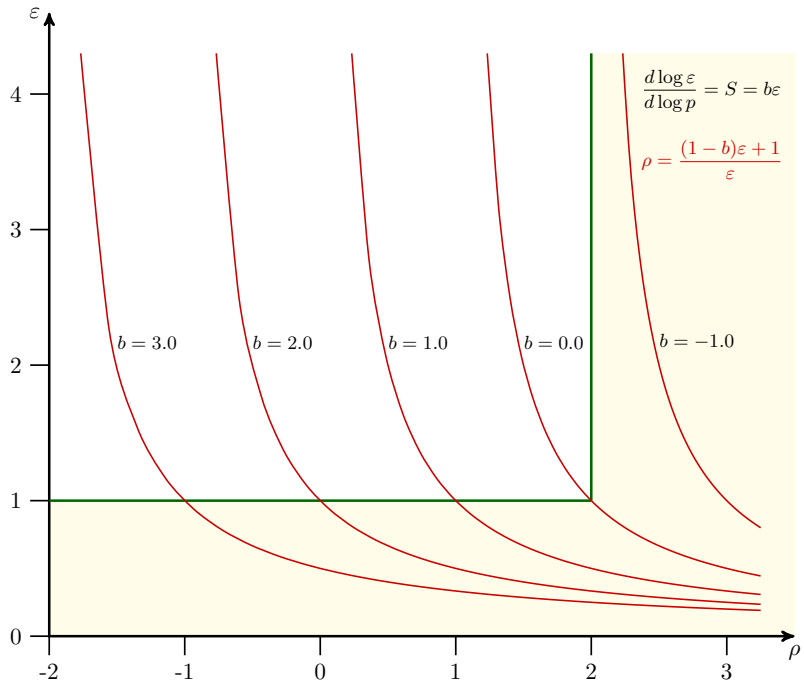
Superelasticity

Figure 8: Superelasticity Manifolds

(a) Iso-Superelasticity



(b) Klenow-Willis

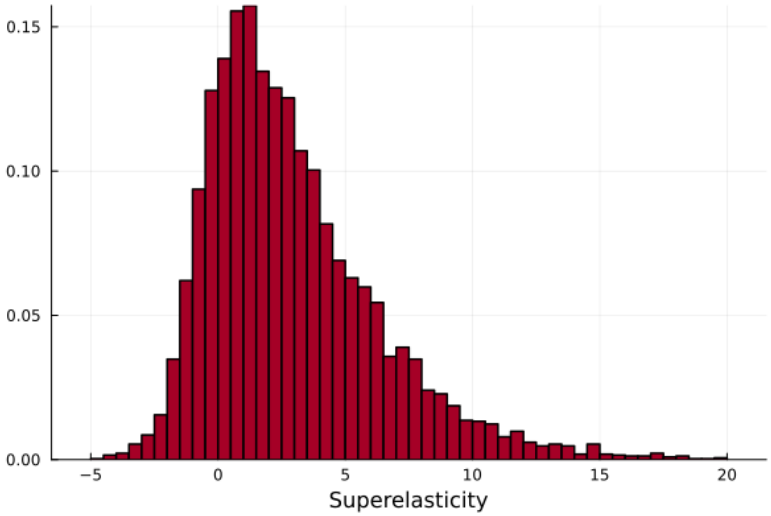


- If $S > 0$ or $b > 0$, demand is subconvex.

Superelasticity

Figure 9: Distribution of Superelasticity Estimates (Cold Cereal)

(a) Superelasticity



(b) K-W Ratio

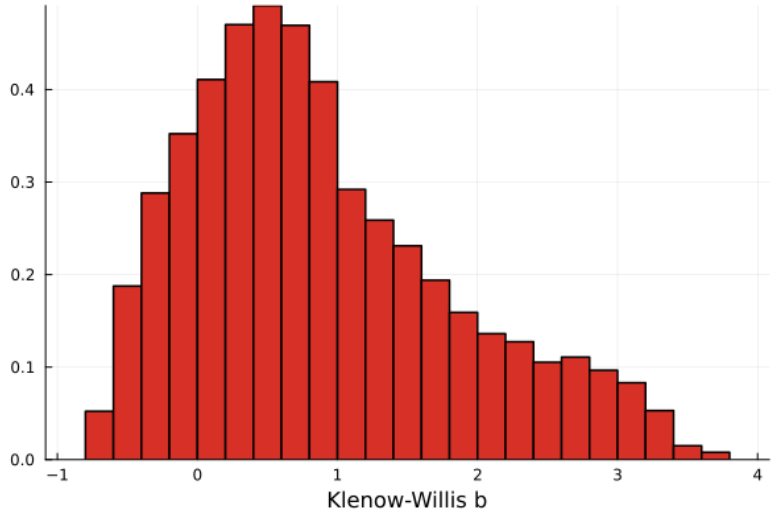


Table 6: Empirical Distribution of Superelasticity (Cold Cereal)

	Superelasticity		Klenow-Willis Ratio		
Elasticity Range	Median	Mean	Median	Mean	% Superconvex
$[4, +)$	4.0766	4.7843	0.7657	0.8963	11.08
$[3, 4)$	2.0214	2.6525	0.5603	0.7240	19.06
$[2, 3)$	1.5200	2.2017	0.5534	0.7851	24.45
$[1, 2)$	1.7351	2.0584	0.9548	1.1431	15.83

Contribution

- To be written.
- .
- .
- .
- .
- .

Thank you!

www.eugeniomiravete.com