A Model of Leveraged Bubbles

Nina Biljanovska[1] Jordi Galí[2] Lucyna Górnicka[1] Alexandros P. Vardoulakis[3]

[1] IMF

[2] CREI, UPF, BSE [3] FRB

CRETE

July, 2026

Disclaimer: The views expressed are those of the authors and do not necessarily represent those of International Monetary Fund, the Federal Reserve Board of Governors or anyone in the Federal Reserve System.

Motivation: Bubbles & Recessions

Jordà et al. 2015; Dell'Aricchia et al. 2011; Greenwood et al. 2020

Empirical facts:

1. Asset-price booms are not always accompanied by elevated credit growth.
2. Small real effects of bursting asset-price booms absent elevated credit.
3. Credit expansion & asset price booms $\rightarrow$ even deeper (2x more), longer recessions when prices burst.

Leveraged Bubbles

Call constellation of elevated credit and prices a *leveraged bubble*, following Jordà et al. (2015), vis-à-vis an *unleveraged bubble* when there is only an asset-price boom.

This paper

What we do

- Develop a simple model to capture and study the real effects of leveraged bubble dynamics, consistent with empirical findings→ emphasis on non-linearities.
- Characterize optimal policy to address bubble externalities on top of usual pecuniary externalities.

How we do it

- Augment Bianchi and Mendoza (2018, JPE) macro model—featuring occasionally binding borrowing constraints—with a stochastic rational asset price bubble.
  - The bubble **grows endogenously** and raises collateral values when it expands, relaxing borrowing limits.
  - When the bubble **bursts**, collateral value collapses, borrowing constraints tighten, and deleveraging amplifies the downturn.
- Calibrate the model to match bubble facts in Jordà, Schularick, and Taylor (2015, JME) and solve for the global non-linear dynamics of bubbles and credit.

Preview of results

Positive results

- **Leveraged vs. unleveraged downturns:** output declines are about **1.5 times larger** when high collateral values coincide with high debt accumulation.
- **Nonlinear amplification:** when a bubble collapse coincides with a negative productivity shock, output falls by **about 11.7%**, compared to **4%** for a bubble burst alone and **6.1%** for a productivity shock alone.
- **Persistence:** leveraged-bubble recessions feature slower recoveries due to prolonged binding borrowing constraints.

Normative results

- **Two externalities:** agents do not internalize how borrowing affects (i) the price of capital and (ii) the growth path of the bubble, and hence collateral values and the tightness of the borrowing constraint.
- **Optimal policy:** a Pigouvian borrowing tax addresses externalities and substantially **mitigates severe recessions**.
- **Welfare gains:** optimal policy delivers welfare gains of about **0.3% of permanent consumption**.

Related Literature

1. Borrowing constraints and endogenous financial crises

Kiyotaki and Moore (1997); Lorenzoni (2008); Mendoza (2010); Geanokoplos (2010); Jermann and Quadrini (2012); Brunnermeier and Sannikov (2014)

- We explore the effect of leveraged and unleveraged bubbles on output, and consider downturns without exogenous tightening of financial conditions.

2. Rational bubble macro models

Kocherlakota (2009); Farhi and Tirole (2012); Martin and Ventura (2012); Galí (2014); Miao and Wang (2018); Biswas et al. (2020); Allen et al. (2022)

- We focus on non-linear joint dynamics of bubbles and credit rather than on long-run growth, steady-state, or linearized dynamics.
- We solve the model globally to match stylized facts of (bursting) leveraged bubbles and derive the optimal policy.

3. Optimal macroprudential policy

Bianchi (2011); Benigno et al. (2013); Bianchi and Mendoza (2018); Davila and Korinek (2018); Jeanne and Korinek (2020)

- We characterize bubble externalities separately from fundamental-price pecuniary externalities.

Economy

- Small open economy with an infinitely-lived representative agent.
  - Owns firm that produces a consumption good using labor, capital, and intermediate inputs.
- The agent can borrow externally by pledging capital and bubble holdings as collateral.
  - World interest rate and pledgeability of collateral are exogenously fixed.
- Capital is in aggregate fixed supply of 1.
- The bubble does not yield dividends but relaxes borrowing constraints.

Bubble Process

- The bubble follows a two-state Markov process governed by indicator variable $\mathcal{I}_t$.
  - Bubble survives with probability π; otherwise it bursts and a new bubble emerges.

$$q_t^b = \begin{cases} u & \text{if } \mathcal{I}_t = 0 \quad \text{w.p.} \quad 1-\pi \\ b_t & \text{if } \mathcal{I}_t = 1 \quad \text{w.p.} \quad \pi \end{cases}$$

- Every new bubble is in aggregate fixed supply of 1.
  - New bubbles are distributed equally among all agents at the beginning of the period they emerge.
  - New bubbles are always exogenously valued at the same price u (could be relaxed).
- If a previous-period bubble survives, its price b_t grows endogenously with economic conditions in a way consistent with rational expectations.
- The bubble can exist in equilibrium despite the infinite-lived representative agent because it relaxes borrowing constraints.

Representative agent's problem

$$\max_{c_t, l_t, v_t, L_{t+1}, k_{t+1}, x_{t+1}} E_0 \sum_{t=0}^{\infty} \beta^t U(c_t - G(l_t))$$

s.t. **Budget constraint:**

$$c_t \leq \underbrace{y_t}_{\text{Output}} - \underbrace{p^v v_t}_{\text{Intermediate good purchases}} + \underbrace{\frac{L_{t+1}}{R} - L_t}_{\text{Net borrowing}} - \underbrace{q_t^k (k_{t+1} - k_t)}_{\text{Net capital purchases}} - \underbrace{q_t^b (x_{t+1} - 1 - (x_t - 1)\mathcal{I}_t)}_{\text{Net bubble purchases}}$$

Borrowing constraint:

$$L_{t+1}/R + \theta p^v v_t \leq m \left(q_t^k k_t + q_t^b (1 + (x_t - 1)\mathcal{I}_t) \right)$$

- Output $y_t = z_t k_t^{\alpha_k} v_t^{\alpha_v} l_t^{\alpha_l}$; k_t: capital, v_t: intermediate good; l_t: labor.
- L_{t+1}: new borrowing; R: interest rate.
- q_t^k and q_t^b: price of capital and bubble ; x_{t+1}: new bubble holdings.
- m: pledgeable portion of capital and the bubble; θ: working capital funding.

Recursive Equilibrium & Bubble Growth

- **State variables.** Absent a bubble the state variables are (z_t, L_t); with the bubble additional states arise $(\mathcal{I}_t, q_t^b)$.
- **Indeterminacy.** The Euler condition for the bubble is consistent with multiple bubble values.

$$q_t^b U_{c,t} = \beta \pi \tilde{E}_t \left(U_{c,t+1} b_{t+1} (1 + m\mu_{t+1}) \right)$$

- **Simplifying assumption.** Zero innovations in the price of the bubble:

$$b_t = \tilde{E}_{t-1}[b_t]$$

- **Bubble growth.**

$$\frac{b_{t+1}}{q_t^b} = \frac{1}{\pi \tilde{E}_t M_{t+1} (1 + m\mu_{t+1})}$$

- Bubble growth affects the entire future path of borrowing constraints.

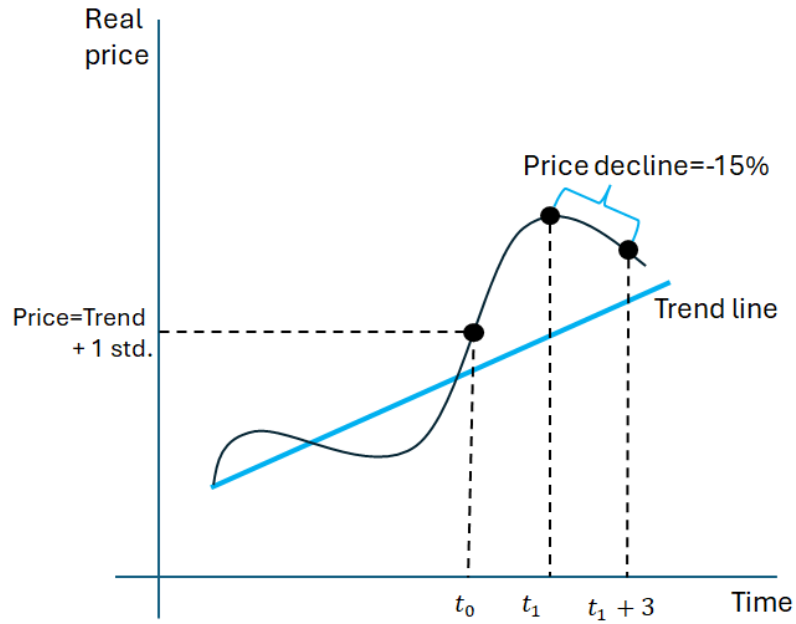
Calibration and Solution Method

Solution Method

- Mendoza and Villalvazo (2020) fixed-point iteration algorithm.
  - Two endogenous states: (L_t, b_t).
- Global solution, occasionally binding constraints.

Calibration

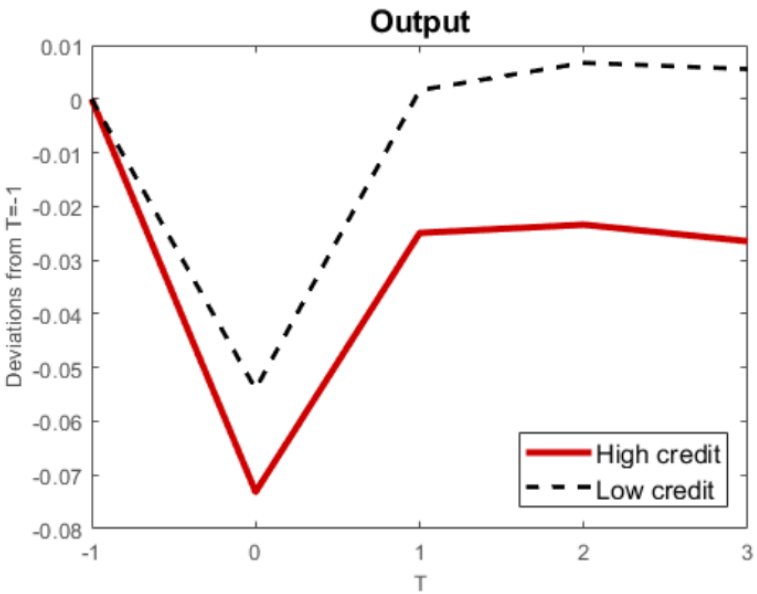
- Baseline parameters: Bianchi and Mendoza (2018).
- Bubble process $\rightarrow$ Match stylized facts in Jorda, Schularick, and Taylor (2015):
  - Identify JST bubble episodes in simulated economy.
  - Target Duration = 3.2 yrs and Amplitude = 17.3%.
  - Set $u = 0.002$ (new bubble value) and $\pi = 0.67$ (survival probability).

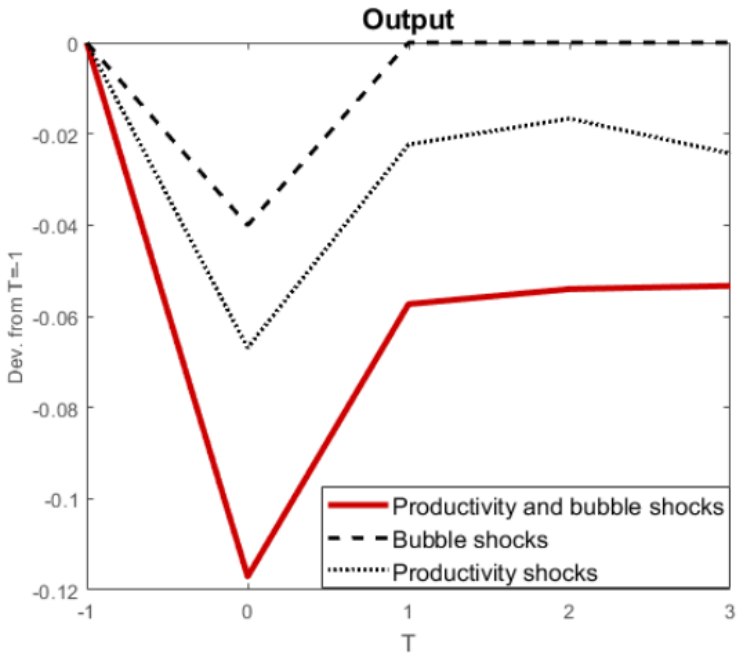


JST Bubble Episode:

- *Start* (t_0): $q_t > \bar{q} + \sigma(q)$.
- *End* (t_1): 15% decline over 3 years.

Debt levels, type of recessions, and bubble crashes





- Recessions following leveraged bubbles are roughly 1.5 times deeper.
- Recoveries are slower when debt accumulation is high.

- Output losses: Bubble (-4%) + Productivity (-6.1%) $<$ Both together (-11.7%).
- Bursting leveraged bubbles + fundamental weakness = severe and persistent recessions.

Optimal Policy: Correcting Market Failures

Externalities Private Agents Don't Internalize

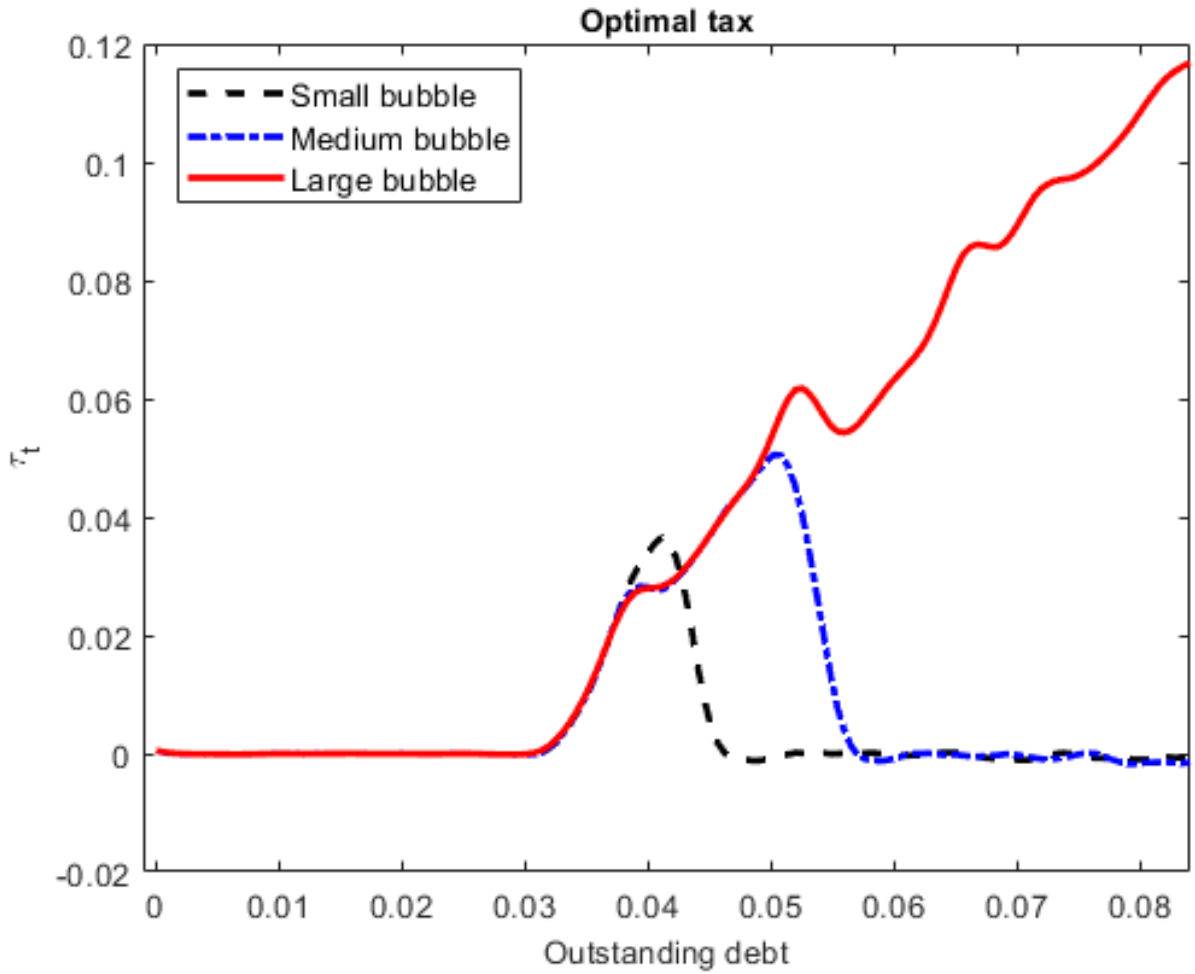
Capital price externality: Borrowing affects q_t^k and $q_{t+1}^k \rightarrow$ influences tightness of current and next-period constraints only.

Bubble growth externality (new): Borrowing affects growth rate of bubble $\rightarrow$ influences future bubble state $\rightarrow$ impacts entire future path of constraints until bubble bursts.

Implementation of time-consistent planner soltuion: Borrowing Tax

$$\tau_t = \underbrace{\text{Bubble growth externality}}_{\text{New externality}} + \underbrace{\text{Fundamental price externality}}_{\text{Standard}}$$

Optimal Tax



- Tax increases with debt and bubble size.
- Declines when borrowing constraint binds contemporaneously.

Conclusion

- Recessions following leveraged asset price bubbles are deeper and more persistent than those following unleveraged booms.
- Time-consistent optimal policy substantially reduces the effect of leveraged-bubble recessions.

Back-up slides

Euler Equations

- **Borrowing decision:**

$$U_{c,t}(1-\mu_t) = \beta R E_t U_{c,t+1}$$

Stochastic discount factor lower than $1/R$.

- **Price of capital:**

$$q_t^k U_{c,t} = \beta E_t U_{c,t+1} \left(F_{k,t+1} + q_{t+1}^k (1 + m\mu_{t+1}) \right)$$

Discounted dividends and future price of capital incorporating collateral premium.

- **Bubble growth:**

$$q_t^b U_{c,t} = \beta \pi \tilde{E}_t \left(U_{c,t+1} b_{t+1} (1 + m\mu_{t+1}) \right)$$

Discounted dividends and future price of capital incorporating collateral premium.

Bubble existence

- A rational bubble needs to satisfy the Euler condition, i.e., being priced appropriately in equilibrium.
- But it also needs to satisfy the transversality condition

$$\lim_{T\to\infty} E_t \beta^T U_{c,t+T} b_{t+T} x_{t+T} = \lim_{T\to\infty} E_t \beta^T U_{c,t+T} b_{t+T} = 0.$$

- Iterating the Euler equation we get

$$q_t^b = \lim_{T\to\infty} E_t \beta^T \Pi_{i=1}^T (1 + m\mu_{t+i}) \frac{U_{c,t+T}}{U_{c,t}} b_{t+T}.$$

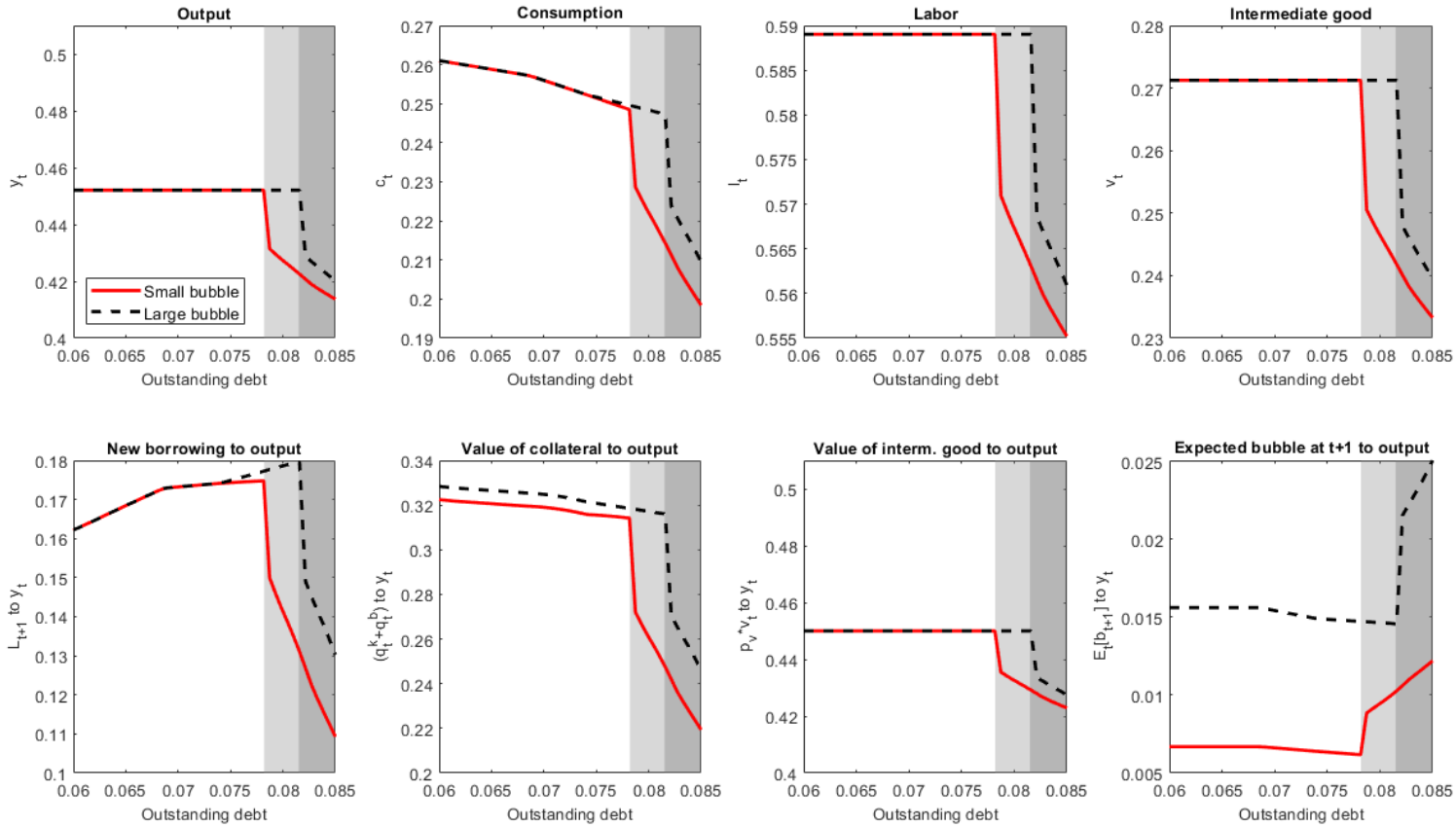
- A rational bubble can exist if the borrowing constraint binds infinitely often, $\lim_{t\to\infty} sup \mu_t > 0$.
- This is true for sufficiently low m.

Summary statistics: Laissez-faire versus planner's solution

	Laissez-faire		**Optimal policy**	
PANEL A	mean	st. dev	mean	st. dev
Output (GDP)		2.5%		2.3%
Consumption		1.7%		1.3%
Credit growth	0.7%	11.4%	0.1%	4.7%
PANEL B				
Incidence of a recession	20.3%		30.2%	
Output decline in a recession	-6.5%		-3.3%	
Incidence of a severe recession	4.4%		0.03%	
Output decline in a severe recession	-11.3%		-13.2%	
PANEL C				
Tax on borrowing			2.2%	1.3%
Welfare gains			0.3%	

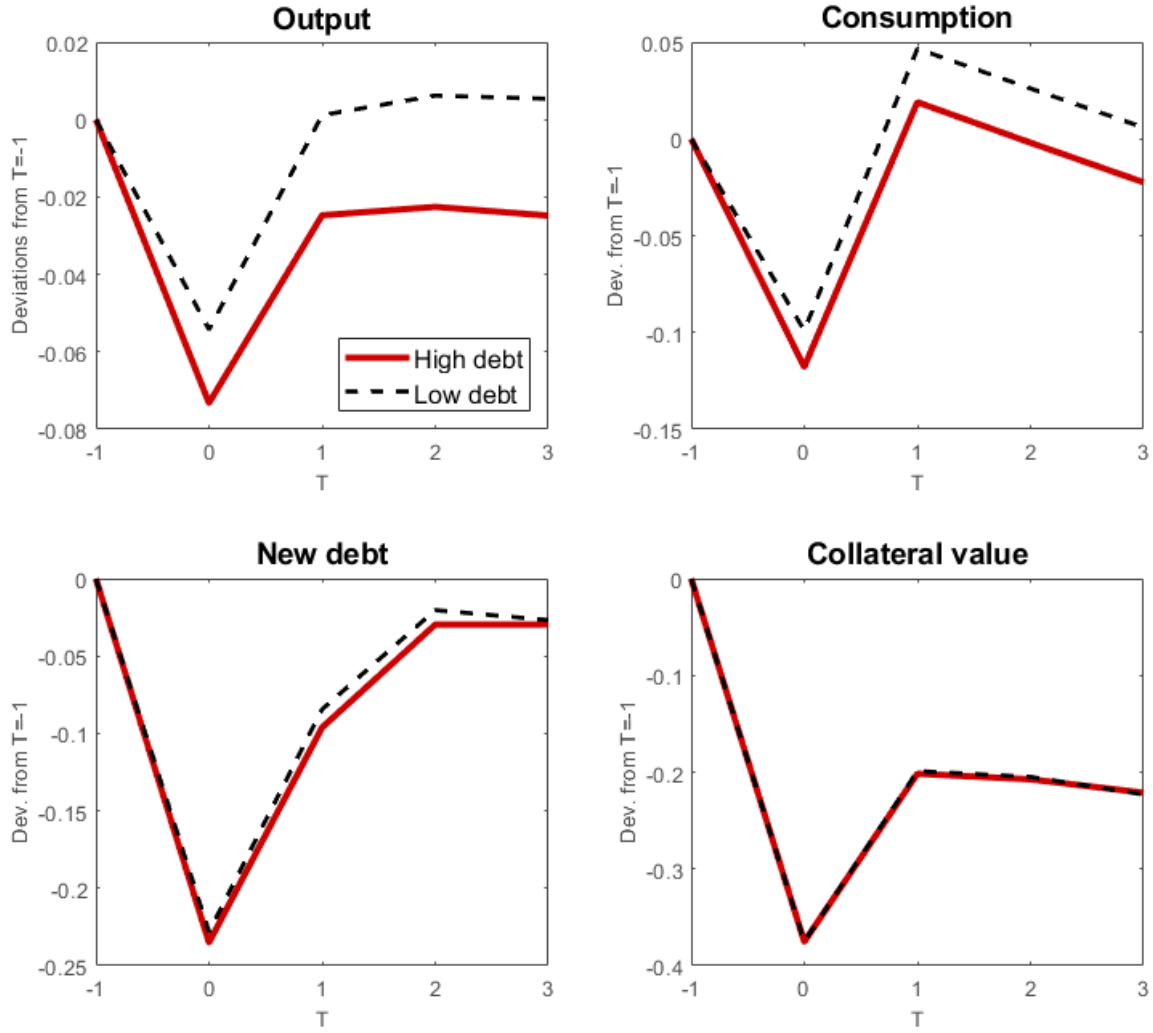
Notes: A recession is defined as a period with an output decline compared to the period before. A severe recession is a recession with an output decline of -10% or more.

Policy Functions



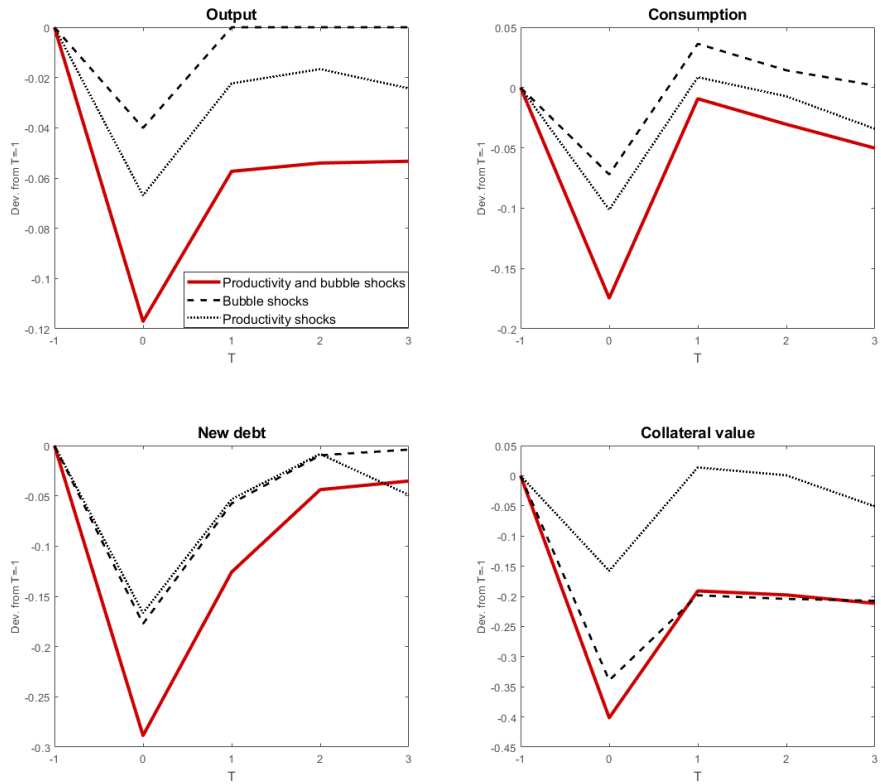
- Larger bubbles support higher debt without triggering the borrowing constraint.
- Binding constraint: consumption, intermediate inputs, labor, and output fall.
- Expected future bubble value increases nonlinearly once the constraint binds.

Event Analysis: The Aftermath of Bubble Crashes



- Recessions following leveraged bubbles are roughly 1.5 times deeper.
- Recoveries are slower when debt accumulation is high.

Types of Recessions: The Role of Bubbles and Fundamentals



- Output losses: Bubble (-4%) + Productivity (-6.1%) $<$ Both together (-11.7%).
- Bursting leveraged bubbles + fundamental weakness = severe and persistent recessions.