Limits of Arbitrage with Optimal Contracts

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Intermediaries in Capital Markets

- Prices in capital markets are determined to a large extent by financial intermediaries rather than households.
- Role of financial intermediaries is increasingly acknowledged in:
  - Finance. Intermediary-based asset pricing.
  - Macroeconomics.
  - International macroeconomics.
- Key themes:
  - Intermediaries absorb imbalances between demand and supply.
    - In doing so, they bring prices closer to fundamentals.
  - Intermediaries have limited risk-absorption capacity.
    - Their capacity is low when they have low wealth or poor performance.

Example: Term Structure of Interest Rates

- Assume decrease in demand for long-term bonds, with no change to current and future expected short rates.

- Impact on term structure:
  - Intermediaries must accommodate decrease in demand by buying long-term bonds.
  - Prices of long-term bonds must decrease ($\rightarrow$ long rates must increase) to induce intermediaries to buy.
  - Impact is large when intermediaries have low risk-absorption capacity.

  Vayanos-Vila (ECMA 2021).

- Time-varying risk premia.
  - Impact on term structure is through risk premia.
  - Prices move without change in "fundamentals."

- Similar mechanisms have been explored in other markets:
  - Stocks. *Greenwood (JFE 2005), He-Krishnamurthy (RES 2012, AER 2013)*
  - Currencies. *Gabaix-Maggiori (QJE 2014), Itskokhi-Mukhin (JPE 2021)*
  - Derivatives. *Garleanu-Pedersen (RFS 2011)*
  - Mortgage-backed securities. *Gabaix-Krishnamurthy-Vigneron (JF 2007)*

Intermediation Frictions

- For intermediaries to matter, they must face frictions in raising capital from households.
  - In the absence of frictions, intermediary-based models reduce to standard models with households. Representative agent, etc.

- Examples of assumed frictions:
  - Intermediaries can raise no equity capital from households.
    - External finance can only be in the form of debt.

    Grossman-Miller (JF 1988), Geanakoplos (1997), Gromb-Vayanos (JFE 2002), Brunnermeier-Sannikov (AER 2014)
  - Equity capital that households invest with intermediaries is exogenous increasing function of intermediaries' past performance.

    Shleifer-Vishny (JF 1997)
  - Equity capital is determined by static contracting even though relationship is dynamic.

    He-Krishnamurthy (RES 2012, AER 2013)

- Strong assumptions on contracting.
  - Emphasis is on equilibrium asset prices rather than on contracting.

Contracting Model

- Optimal contracting between intermediaries (agents) and households (principals).
  - No restriction to debt.
- Dynamic aspects are important.
  - Poor performance $\rightarrow$ Principals invest less capital with agents.
- Dynamic financial contracting.
  - Agency arises from moral hazard. Agent can "steal."
  - Agent can be rewarded using money or future continuation utility.
  - Used in corporate finance to study firms' capital structure and investment decisions.

DeMarzo-Sannikov (JF 2006), Biais-Mariotti-Platin-Rochet (RES 2007), DeMarzo-Fishman (RFS 2007).

This Paper

- Intermediaries raise capital from households using optimal contracts.
- Use dynamic financial contracting, bringing it into equilibrium asset pricing.
  - Two-way feedback between contracts and equilibrium asset prices.

Main Results

- Effectiveness of arbitrage depends on size of imbalances between demand and supply.
  - Imbalances are absorbed with small price impact when they are small.
  - Imbalances have large price impact and can generate amplification when they are large.
- Longer-term contracts between principals and agents result in more effective arbitrage.
  - Lower price impact.
  - Lower price volatility feeds back into better contracting.
- Possible implications for contracts in asset management.
  - Should longer-term contracts be encouraged?

Baseline Model

Assets

- Three periods $t = 0, 1, 2$.
- Riskless asset, zero net return.
- Risky asset.
  - Dividend D_2 in period 2, distributed in $[0, \infty)$ with continuous density $f_2(D_2)$.
  - Price S_t in period $t = 0, 1$.
  - Supply $\theta \geq 0$ shares.

Agents

- Direct traders.
- Delegated traders.
  - Pairs of principals and agents.
- Noise traders.
  - Supply u_t shares in period $t = 0, 1$.
  - u_0 deterministic.
  - u_1 distributed in $[0, \infty)$ with continuous density $f(u_1)$.
    - Imbalance. What will be its price impact?
  - Noise traders can be folded into direct traders for a welfare analysis.

Direct Traders

- Competitive, continuum with measure one.
- Risk-neutral with utility

$$W_{2d} - \frac{\alpha_{0d}}{2} I_{0d}^2 - \frac{\alpha_{1d}}{2} I_{1d}^2.$$

  over wealth W_{2d} in period 2 and investment $I_{td} \geq 0$ in risky asset in period $t = 0, 1$.
  - Quadratic terms are reduced form for risk aversion.
  - α_{td} is risk-aversion coefficient in period $t = 0, 1$.
- Endowed with asset supply θ in period 0.

Delegated Traders

- Pairs of principals and agents. Competitive, continuum with measure one.

- Agents.
  - Hold risky asset in a fund on behalf of principals.
  - Observe asset prices and dividend.
  - Moral hazard: Can reduce fund value and divert some of the money.
    - Reduce fund value by $X \geq 0 \rightarrow$ Divert ϵX, where $\epsilon \in [0, 1]$.
    - No agency when $\epsilon = 0$.
    - Diversion can occur in periods $t = 1, 2$.

- Principals.
  - Do not observe asset prices and dividend. Observe certified reports of fund value post-diversion.
    - Reports are observed in periods $t = 1, 2$.
  - Risk-neutral with utility
    $$W_2 - \frac{\alpha_0}{2} I_0^2 - \frac{\alpha_1}{2} I_1^2$$
    over wealth W_2 in period 2 and investment $I_t \geq 0$ in period $t = 0, 1$.

Contracts

- Contracts are written in period 0 and can depend on reports of fund value in periods $t = 1, 2$.

- Contracting variables:
  - Investment $I_0 \geq 0$ in period 0.
  - Investment $I_1 \geq 0$ in period 1.
    - Can depend on report $\hat{V}_1$ about fund value V_1 in period 1, or equivalently on report $\hat{u}_1$ about u_1.
  - Cash payment $C_2 \geq 0$ to agent in period 2.
    - Can depend on reports $(\hat{V}_1, \hat{V}_2)$ about fund value (V_1, V_2) in periods 1 and 2, or equivalently on $\hat{u}_1$ and on report $\hat{D}_2$ about D_2.

Equilibrium in Period 1

Optimization by a Direct Trader

- Direct trader's utility in period 1 is

$$\underbrace{\theta S_0 + I_{0d}\frac{S_1 - S_0}{S_0} + I_{1d}\frac{\mathbb{E}(D_2) - S_1}{S_1}}_{\text{expected wealth}} - \frac{\alpha_{0d}}{2} I_{0d}^2 - \frac{\alpha_{1d}}{2} I_{1d}^2.$$

- First-order condition is

$$I_{1d} = \frac{\mathbb{E}(D_2) - S_1}{\alpha_{1d} S_1}.$$

- Combining with market clearing

$$I_{1d} + I_1 = (\theta + u_1) S_1,$$

  yields equilibrium price in period 1.

- Assume S_1 decreasing in u_1. Verify that this holds in equilibrium.

Incentive Constraint in Period 2

- C_2 must rise with $\hat{V}_2$ to deter under-reporting. Constraint is binding, implying

$$C_2(\hat{V}_1, \hat{V}_2) = \epsilon \hat{V}_2 + \underline{C}_2(\hat{V}_1).$$

  Cash payment consists of
  - Performance fee $\epsilon \hat{V}_2$.
  - Base payment $\underline{C}_2(\hat{V}_1)$. Can be interpreted as fee in period 1.

- Since in equilibrium $\hat{V}_1 = V_1$ and $\hat{V}_2 = V_2 = I_1(V_1)\frac{D_2}{S_1}$, agent's utility in period 1 is

$$U_{1a}(V_1) = I_1(V_1)\frac{\mathbb{E}(D_2)}{S_1} + \underline{C}_2(V_1).$$

  - Utility increases in $I_1(V_1)$ (fund's AUM).
  - Can compensate agent with AUM in period 1 ($\rightarrow$ fee in period 2) and fee in period 1.

- Contract choice reduces to choosing $(I_0, I_1(u_1), \underline{C}_2(u_1))$. Take I_0 as given for now.

Optimization by a Principal

- Principal's utility in period 0 is

$$\underbrace{\mathbb{E}\left[I_0\frac{S_1(u_1)-S_0}{S_0}+I_1(u_1)\frac{\mathbb{E}(D_2)-S_1(u_1)}{S_1(u_1)}\right]}_{\text{expected wealth gross of agent's utility}}$$
$$-\frac{\alpha_0}{2}I_0^2-\frac{\alpha_1}{2}\mathbb{E}\left[(I_1(u_1))^2\right]-\underbrace{\mathbb{E}\left[I_1(u_1)\frac{\mathbb{E}(D_2)}{S_1(u_1)}+\underline{C}_2(u_1)\right]}_{\text{agent's utility}}.$$

- Principal maximizes utility subject to agent's incentive constraint in period 1 (IC1)

$$\underbrace{I_1(u_1)\frac{\mathbb{E}(D_2)}{S_1(u_1)}+\underline{C}_2(u_1)}_{\substack{\text{expected utility}\\ \text{for report } u_1}}\geq\underbrace{\epsilon I_0\frac{S_1(u_1)-S_1(\hat{u}_1)}{S_0}}_{\text{diversion proceeds}}+\underbrace{\epsilon I_1(\hat{u}_1)\frac{\mathbb{E}(D_2)}{S_1(u_1)}+\underline{C}_2(\hat{u}_1)}_{\substack{\text{expected utility}\\ \text{for report } \hat{u}_1}},$$

for all u_1 and $\hat{u}_1 \geq u_1$.

Key Trade-Off

- In the absence of (IC1), principal sets $\underline{C}_2(u_1) = 0$ and

$$\eta_1(u_1) = \frac{(1-\epsilon)\mathbb{E}(D_2) - S_1(u_1)}{\alpha_1 S_1(u_1)} \equiv I_1^{*P}(u_1).$$

  - $\eta_1^{*P}(u_1)$ is smaller than first-best (FB) value

$$\frac{\mathbb{E}(D_2) - S_1(u_1)}{\alpha_1 S_1(u_1)} \equiv I_1^{*}(u_1),$$

    which maximizes sum of utilities of principal and agent, because higher $I_1(u_1)$ raises agent's compensation.

- If principal sets $I_1(u_1) = I_1^{*P}(u_1)$ for all u_1, then must raise $\underline{C}_2(u_1)$ for low values of u_1 (high values of S_1) to deter diversion by agent.

Carrots

- Principal economizes on $\underline{C}_2(u_1)$ by lowering $I_1(u_1)$ for low values of u_1.

Sticks

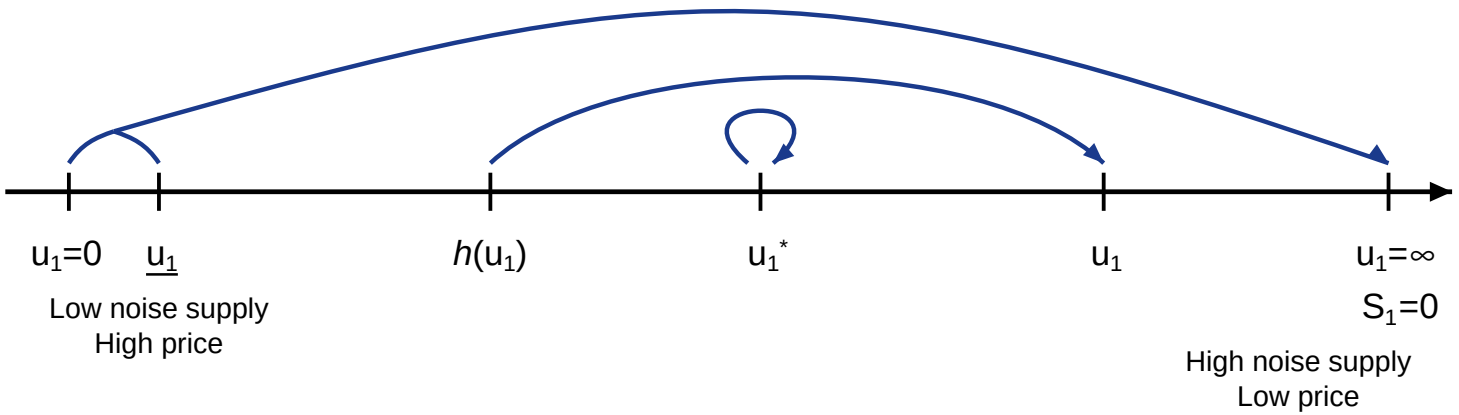
Sorting Property

- (IC1) is two-dimensional: $u_1 \in [0, \infty)$ and $\hat{u}_1 \in [u_1, \infty)$.
  - For which values is (IC1) binding?
  - Is local (IC1) binding, as is common in mechanism design settings?
- Sorting Property: Stronger incentive to divert when agent observes lower u_1.
- Intuition: Agents with lower u_1 have worse investment opportunities $\rightarrow$ Less affected when principal cuts investment.

Relaxed Problem

- Assume (IC1) only between $h(u_1)$ and u_1, where $u_1 \in [u_1^*, \infty)$ and $h(u_1)$ is a decreasing function from $[u_1^*, \infty)$ to $(\underline{u}_1, u_1^*]$.

- Agent observing $h(u_1)$ must prefer
  - To report $h(u_1)$ truthfully.
  - Than to divert and report $u_1 > h(u_1)$.



- (IC1) becomes one-dimensional, but not local.

Solution of Relaxed Problem

- Sticks: For all $u_1 \in [u_1^*, \infty)$,
  - $I_1(u_1) < I_1^{*P}(u_1)$.
  - $\underline{C}_2(u_1) = 0$.

- Carrots: For all $u_1 \in [0, u_1^*)$,
  - $I_1(u_1) > I_1^{*P}(u_1)$.
  - $I_1(u_1) = I_1^*(u_1)$ when $\underline{C}_2(u_1) > 0$.
    - Compensate with cash rather than AUM when $I_1(u_1)$ reaches $I_1^*(u_1)$.

- First-order condition (ODE1) wrt $I_1(u_1)$ for $u_1 \in [u_1^*, \infty)$:
  - When $\underline{C}_2(h(u_1)) = 0$,

$$\underbrace{\left(I_1^{*P}(u_1) - I_1(u_1)\right) f(u_1)}_{\text{benefit of raising } I_1(u_1)} = \underbrace{\left(I_1^{*P}(h(u_1)) - I_1(h(u_1))\right) f(h(u_1))h'(u_1)}_{\text{cost of raising } I_1(h(u_1))}.$$

  - When $\underline{C}_2(h(u_1)) > 0$,

$$\underbrace{\left(I_1^{*P}(u_1) - I_1(u_1)\right) f(u_1)}_{\text{benefit of raising } I_1(u_1)} = \underbrace{-\frac{\epsilon \mathbb{E}(D_2)}{\alpha_1 S_1(h(u_1))} f(h(u_1))h'(u_1)}_{\text{cost of raising } \underline{C}_2(h(u_1))}.$$

Determination of $h(u_1)$

- Solution of relaxed problem has same general form regardless of $h(u_1)$.
- $h(u_1)$ can be chosen so that solution of relaxed problem satisfies (IC1) for all $(u_1, \hat{u}_1)$.
- **Proposition:** Solution of relaxed problem satisfies (IC1) for all $(u_1, \hat{u}_1)$ if reporting u_1 is locally optimal for agent observing $h(u_1)$.
  - Since local optimality requires that $\hat{u}_1 = u_1$ maximizes

$$\epsilon l_0 \frac{S_1(h(u_1)) - S_1(\hat{u}_1)}{S_0} + \epsilon l_1(\hat{u}_1) \frac{\mathbb{E}(D_2)}{S_1(h(u_1))} + \underline{C}_2(\hat{u}_1)$$

and $\underline{C}_2(u_1) = 0$ for $u_1 \in [u_1^*, \infty)$, first-order condition (ODE2) is

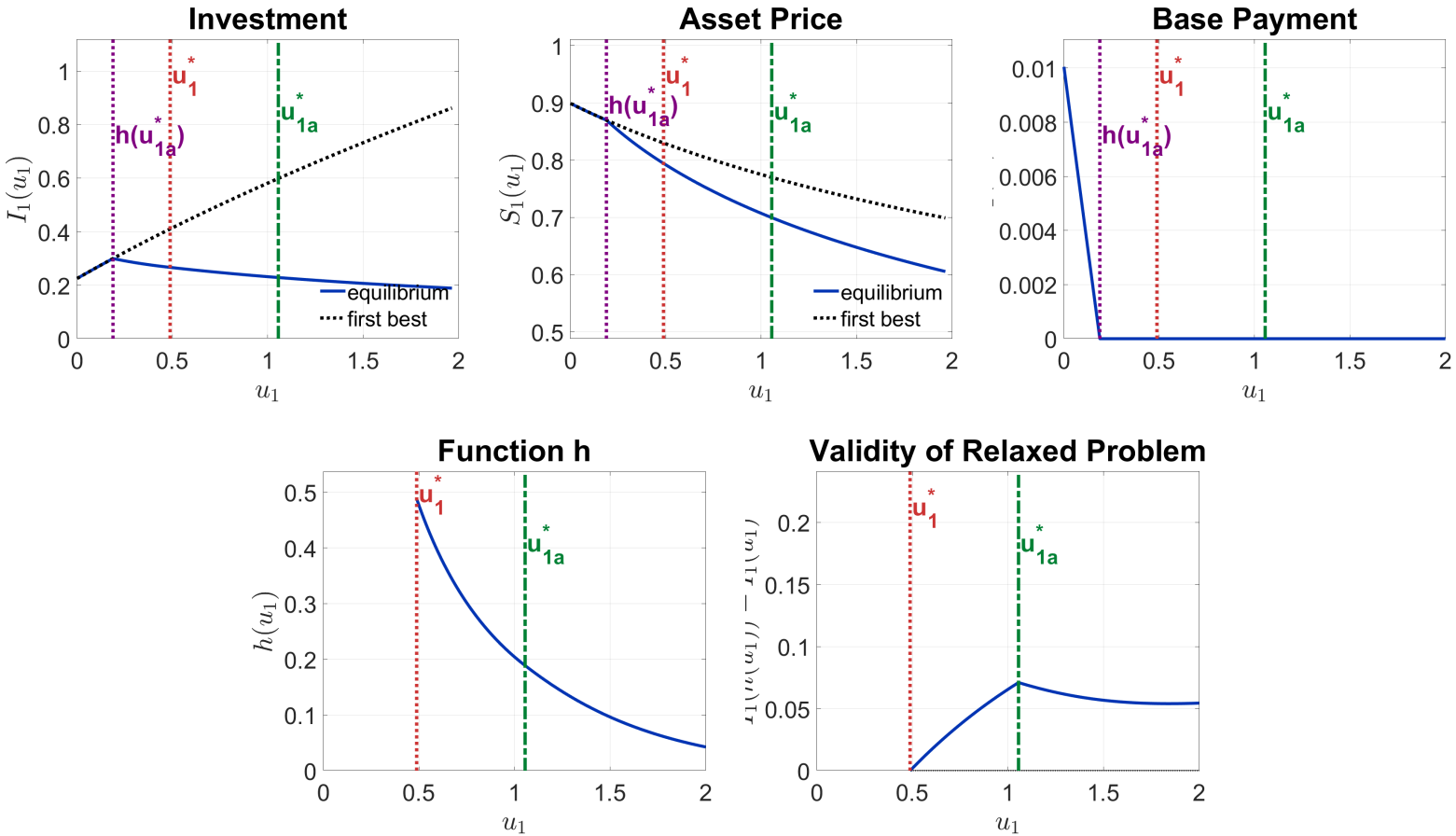
$$-l_0 \frac{S_1'(u_1)}{S_0} + l_1'(u_1) \frac{\mathbb{E}(D_2)}{S_1(h(u_1))} = 0.$$

- Proposition holds if $l_1(h(u_1)) \geq l_1(u_1)$ for all $u_1 \in [u_1^*, \infty)$. Otherwise, solve a modified relaxed problem in which local (IC1) binds for small u_1.

Equilibrium in Period 1

- Equilibrium:
  - $(I_1(u_1), h(u_1))$ solve ODE1 and ODE2 given $S_1(u_1)$.
  - $S_1(u_1)$ is equilibrium price given $I_1(u_1)$.
- Problem reduces to solving ODE system in $(I_1(u_1), h(u_1))$.
- Threshold u_1^* is determined by $h(\infty) = \underline{u}_1$ and by $\underline{u}_1 = 0$ if $I_1(\infty) = 0$.

Equilibrium in Graphs



- Small u_1.
  - Principal's investment $I_1(u_1)$ takes FB value. Increases in u_1.
  - Small price impact.
- Large u_1.
  - Principal's investment significantly below FB value. Decreases in u_1.
  - Large price impact.

Extended Model

Assets

- Add a period 3.
- For now:
  - Risky asset is as in baseline model.
  - New asset can be traded in period 2.
    - Dividend D_3 in period 3, distributed in $[0, \infty)$ with continuous density $f_3(D_3)$.
    - Exogenous deterministic price S_2 in period 2.
- Work in progress:
  - Risky asset pays a dividend D_2 in period 2 and D_3 in period 3.
  - Endogenous price S_2 in period 2 (and in periods 0 and 1).

Contracts

- Contracts are written in period 0 and can depend on reports of fund value in periods $t = 1, 2, 3$.

- Contracting variables:
  - Investment $I_0 \geq 0$ in period 0.
  - Investment $I_1 \geq 0$ in period 1.
    - Can depend on report of fund value in period 1, or equivalently on report $\hat{u}_1$ about u_1.
  - Investment $I_2 \geq 0$ in period 2.
    - Can depend on report of fund value in periods 1 and 2, or equivalently on $\hat{u}_1$ and on report $\hat{D}_2$ about D_2.
  - Payment $C_3 \geq 0$ to agent in period 3.
    - Can depend on reports of fund value in periods 1, 2 and 3, or equivalently on $\hat{u}_1$, $\hat{D}_2$ and on report $\hat{D}_3$ about D_3.

Optimization by a Principal in Period 1

- Agent's utility in period 1 is

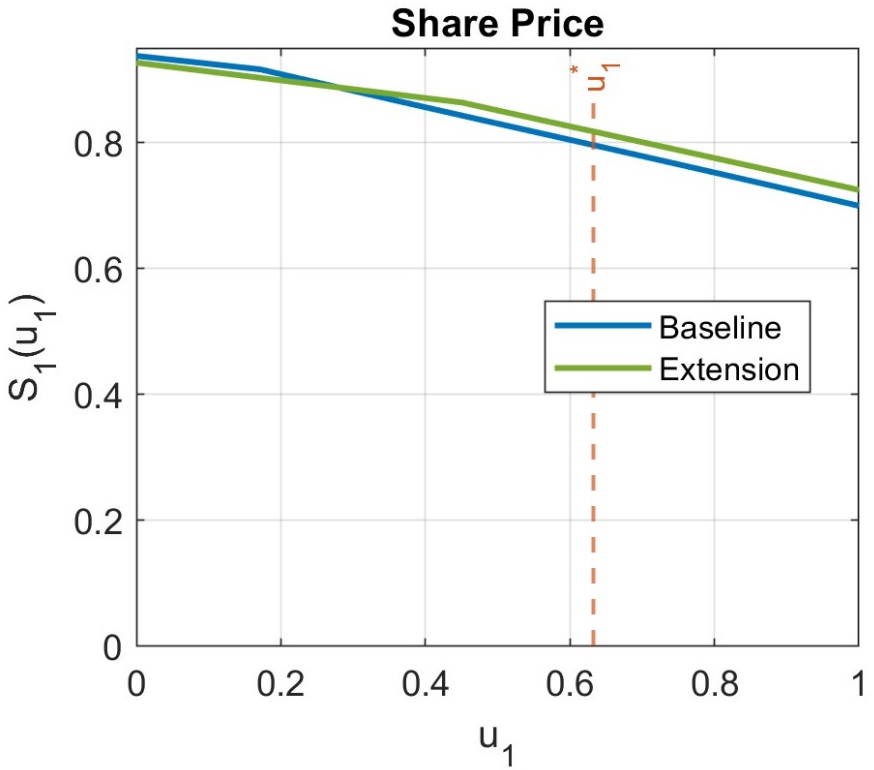
$$U_{1a}(u_1) = \epsilon I_1(u_1)\frac{\mathbb{E}(D_2)}{S_1(u_1)} + \epsilon \underline{I}_2(u_1)\frac{\mathbb{E}(D_3)}{S_2} + \underline{C}_3(u_1),$$

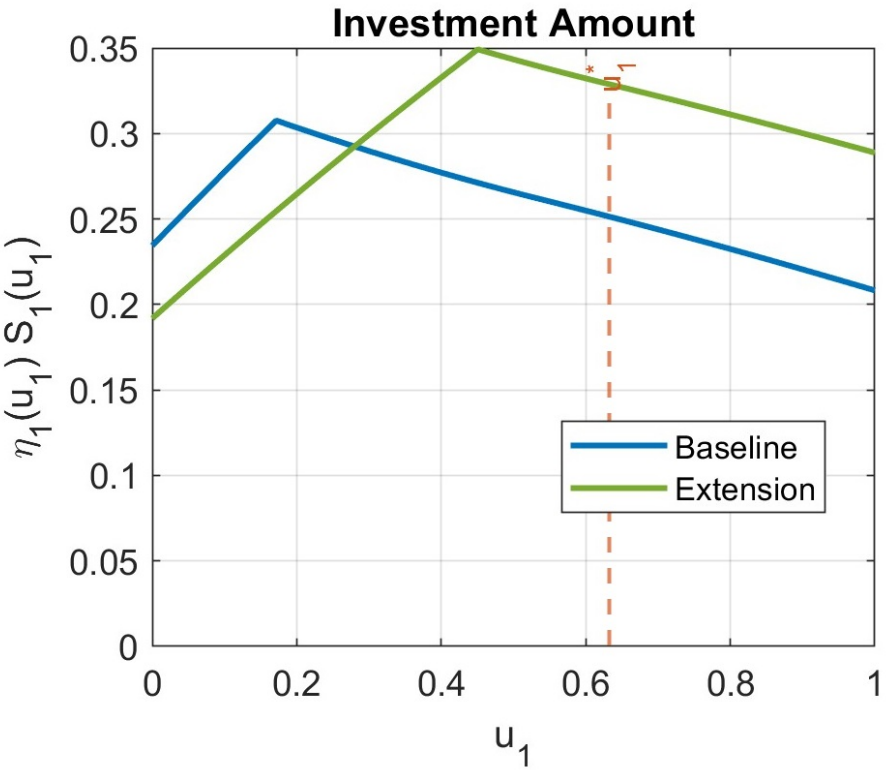
  where $\underline{I}_2(u_1) \equiv I_2(u_1, 0)$ and $\underline{C}_3 \equiv C_3(u_1, 0, 0)$.

- Can compensate agent with AUM in periods 1 and 2 ($\rightarrow$ fee in period 3) and fee in period 1.

- Key trade-off is modified.
  - Sticks: Can reduce not only $I_1(u_1)$ for $u_1 \in [u_1^*, \infty)$, but also $\underline{I}_2(u_1)$.
  - Carrots: Can increase not only $I_1(u_1)$ and $\underline{C}_3(u_1)$ for $u_1 \in [0, u_1^*)$, but also $\underline{I}_2(u_1)$.
  - $\rightarrow$ AUM changes in period 1 are smaller with longer-term contracts.
  - $\rightarrow$ Smaller price impact.

Equilibrium in Graphs





- With long-term contracts, price impact is smaller, especially for large u_1.
- Principal reduces AUM less than with short-term contracts following poor performance in period 1.

Conclusion

- Use dynamic financial contracting to study delegated investment and its asset-pricing implications.
  - Flow-performance relationship is optimal response by principals to agents' moral hazard.
- Main results:
  - Arbitrage is more effective for small shocks than for large shocks.
  - Arbitrage is more effective when contracts between principals and agents are longer-term.