# A Weak-Collusion-Proof Auction for Procurement: Theory and Evidence

Juan Sebastián Ivars and Nikiforos Zampetakis

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# Cartels in Public Procurement

- Public procurement is a relevant activity ($\sim$ 13% GDP OECD Countries).
- Governments often require auctions as the default purchasing methods.
- One of the main threats to auctions: **Collusion**.
- Different agencies provide guidelines to detect and deter collusion in public procurement.
  - OECD (2010, 2025), DOJ Procurement Collusion Strike Force

# Collusion-Proof Auctions

- Different authorities have implemented auctions with a randomization step.
  - One reason is collusion deterrence.
  - E.g. Colombia & Italy, Chile & EU.
- In practice it remains unclear whether randomization actually deters collusion.
- Theory provides conditions under which an auction mechanism is collusion-proof.
  - Laffont and Martimort (1997, 2000); Che and Kim (2006, 2009); Pavlov (2008); Che et al. (2018)

# This Paper

- We focus on bidding rings without transfers (*weak-collusion*).
  - E.g. electrical generators, road construction, Texas school milk market.
- We propose a weak-collusion-proof auction mechanism
  - Show that is strategy-proof & weak-collusion-proof (WCP).
  - Empirically measure its welfare effects in an actual collusive market.
    - Texas school milk market.
- We show most auction formats with randomization are not collusion-proof.

# Literature

## Theory

- Collusion in auctions: Mailath and Zemsky (1991); Aoyagi (2003); Caillaud and Jéhiel (1998); Che and Kim (2009); Dequiedt (2007)

  **This paper:** analyses a static auction without transfer across firms to enforce collusion.

- Weak-Collusion, Cartels and Weak-Collusion-Proof Auctions: McAfee and McMillan (1992); Che et al. (2018)

  **This paper:** interim cartel formation and auction implementability.

- Particular auction designs: Engel and Wambach (2006); Decarolis (2018); Gehrlein et al. (2025)

- Collusion and mechanism design: Laffont and Martimort (1997, 2000)

## Empirical

# Literature

## Theory

## Empirical

- Mechanism design in auctions: Haile (2001); Athey et al. (2013); Roberts and Sweeting (2013); Bolotnyy and Vasserman (2023); Marra (2024); Cuesta and Tebaldi (2025).
  **This paper**: quantifying the outcomes of a WCP mechanism on cartels profits, procurer's costs and welfare.

- Procurement setups and collusion: Pesendorfer (2000); Chassang and Ortner (2019); Starc and Wollmann (2025); Ortner et al. (2025).
  **This paper**: empirical assessment of a theoretically-proven WCP mechanism.

# Theory

# Model - Environment

- Single contract awarded to one bidder, indexed by $i$,
- $N$ bidders, (2 for the presentation),
- Each bidder $i$ has a $c_i \in [\underline{c}, \overline{c}]$ drawn from an i.i.d. cumulative distribution function $F$, smooth, increasing and continuous,
- There is a bidding function, $b : [\underline{c}, \overline{c}] \to \mathbb{R}^+ \cup \{0\}$,
- Sellers are risk-neutral: $u_i(p_i, c_i) = \max\{p_i - c_i, 0\}$,
- Sellers compete in a Reverse Second-Price Auction (Vickrey Procurement Auction): $p_i = \min_{-i} b(c_{-i})$, *(we will modify the auction after)*
- The interim utility of bidder $i$ is denoted by :

$$U_i = \mathbb{E}[u_i(c_i, c_{-i}) \mid c_i] = (p_i - c_i)\ Prob(b(c_i) \leq b(c_{-i}) \mid c_i)$$

# Vickrey Auction is not WCP - Adaptation of Che et al. (2018)

**Vickrey Procurement Auctions are**

1. WCP when the cumulative distribution of the marginal costs is convex
2. Not WCP when the cumulative distribution of the marginal costs is locally concave.

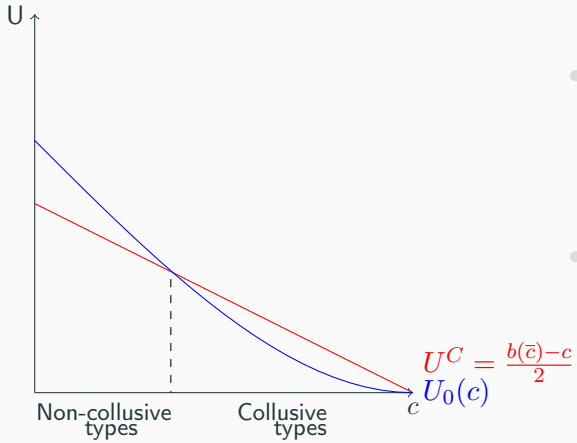
**Intuition**

1. The distribution of marginal costs is convex: *(unravels the cartel)*
   - Few strong types (low costs): they win the auction with high probability,
   - They have someone close with low probability.
2. Non-convex regions: *(better splitting the market)*
   - Lots of strong types (low costs), low probability of winning,
   - The winner finds someone sufficiently close to him after winning.

**Solution to cartel formation:** ironing cost distribution by **randomizing**.

# Profitability of Weak-Collusion - Examples

**Convex distribution - Beta(2,1)**




- Bidders at the left of the intersection are better by not forming the cartel,
- The cartel unravels recursively, by each (potential) member of the cartel, after knowing his type deciding to not to participate,
- With a concave distribution, every bidder prefers to participate in the cartel,

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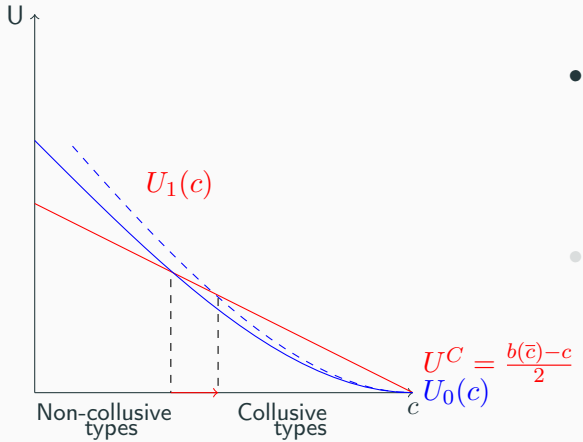


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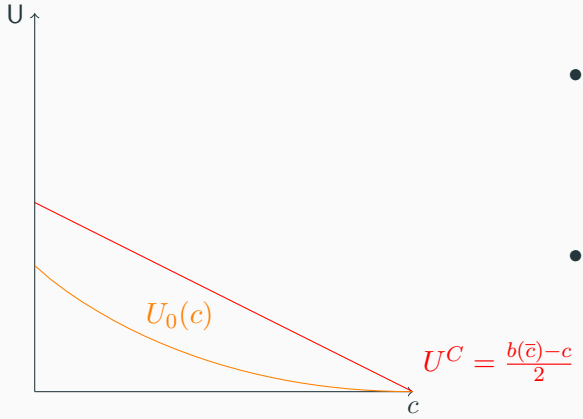
# Profitability of Weak-Collusion - Examples

**Concave distribution - Beta(0.5,1)**

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- With a concave distribution, every bidder prefers to participate in the cartel,

**Assumption 1 (A1).** Auctioneer knows $F_0$.

Assumption

**Second Interval Auction (SIA)**

- Auctioneer convexifies $F_0$,
- From this convexification, the auctioneer gets two types of sets $I$:
  1. Singleton intervals $I = \{c\}$, where $F_0$ is convex,
  2. Intervals $I^P \subseteq [\underline{c}, \overline{c}]$, where $F_0$ was linear or concave.
- Auctioneer **allocates** the contract using these intervals.

# *Second-Interval Auction II*

**In a SIA an auctioneer allocates the good in the following way:**

- Allocates the contract to a bid in the lowest interval, $I_{min}$
- Pays the minimum bid of the second lowest interval, $I_{2min}$

Winning bids and prices paid

| $I_{min}$ / $I_{2min}$ | $I_{2min}$ is a singleton | $I_{2min} \in \mathcal{I}^P$ |
|---|---|---|
| $I_{min}$ is a singleton | $i$ wins, $p_i = \min_j b_{j \neq i}$ | $i$ wins, $p_i = \min_j b_{j \neq i} \in I_{2min}$ |
| $I_{min} \in \mathcal{I}^P$ | $i$ wins with $P = \frac{1}{\eta}$, $p_i = \min_j b_{j \neq i}$ | $i$ wins with $P = \frac{1}{\eta}$, $p_i = \min_j b_{j \neq i} \in I_{2min}$ |

Notes: $i$ represents a bidder in $I_{min}$ while $j$ a bidder in $I_{2min}$, $P$ denotes probability, $\eta$ is the amount of bidders in $I_{min}$.

- If there is only one interval (fully concave distribution), the allocation is fully random and the price paid is the maximum bid.

**Definition 1.** An auction is strategy-proof if for reports $\tilde{c}$ and types $c$:

$$U(b(c_i), b(\tilde{c}_{-i}), c_i) \geq U(b(\tilde{c}_i), b(\tilde{c}_{-i}), c_i), \ \forall c_i.$$

**Definition 2.** An auction is weak-collusion-proof if bidding competitively is better than forming a cartel:

$$U(b(c_i), b(c_{-i}), c_i) \geq U^C(b(\overline{c}), b(\overline{c}), c_i).$$

# Cartel Formation

**When does a cartel form?** *Tension between ex-ante and interim:*

A cartel might be profitable ex-ante (McAfee and McMillan, 1992) but not interim (Che et al., 2018).

- We consider cartel formation at the interim stage.
- Cartel involves no form of commitment, transfer or hard information, it is just an agreement to split the market without transfers.
- Thus, $U^C(b(\overline{c}), b(\overline{c}), c_i) = \frac{b(\overline{c})-c_i}{2}$.

# *Second-Interval Auction III* - Properties (cntd)

**Proposition 1.** *A SIA is strategy-proof and weak-collusion-proof.*

**Intuition**

- Strategy-proof:
  1. When the cost distribution is convex, SIA is simply a Vickrey Procurement Auction,
  2. When the cost distribution is locally concave somewhere, your actual bidding does not affect the interval creation, so your strategy is exactly as before.
- Weak-collusion-proof:
  1. When the cost distribution is convex, the cartel unravels itself, since the strong types do not want to participate
  2. When it is locally concave, their market splitting agreement cannot do better than the randomization device for the types willing to collude.

# Other Results

1. **Randomization before the process (participation) is not WCP**
   - It changes the realized distribution of marginal costs.
   - *Example:* in a S-Shape cost-distribution it can reinforce collusion by excluding the very few strong types.
2. **Randomization after the process (allocation) is not WCP**
   - While affecting the allocation, strong types have even less incentives to defect the cartel, since with some positive probability they will not be awarded.

# Empirical

# Outline

- Application of the SIA mechanism in a market with a busted cartel.
  - Texas school milk market.
- We recover the cost distribution from the bids in the competitive markets.
- Using the estimated cost distribution:
  - We perform a Monte Carlo analysis to compare FPA - FPA under collusion - SIA.
  - We compare the actual collusive bids with simulated bids under SIA.

# Texas Milk Market

- School districts buy milk to provide to the students.
- Contracts for the supply of school milk are allocated through a Reverse First Price Auction (FPA) once a year.
- School districts are heterogeneous (#students, #delivery points etc.)
- Market susceptible to cartelization.
  - Concentrated market, homogeneous product, firms' repeated interaction.
- Well studied cases of bid-rigging cartels busted by DOJ in different states.
  - Porter and Zona (1993); Hewitt et al. (1996); Lanzilioti (1996); Lee (1999); Pesendorfer (2000); Caoui (2022)

# Data

- Collected by DOJ during the time of the investigation.
- School milk contracts awarded between 1980 - 1992.
- 3 Market areas:
  - Collusive area: Dallas-Fort Worth.
  - Competitive areas: Waco and San Antonio.
- We observe
  - the winning bid, the other bids, cost components, contractual demands (cooler, #deliveries etc.), escalated bid, the winner's identity and the distance of each seller's closest plant to the school district.
- 1,034 auctions and 3,496 bids.

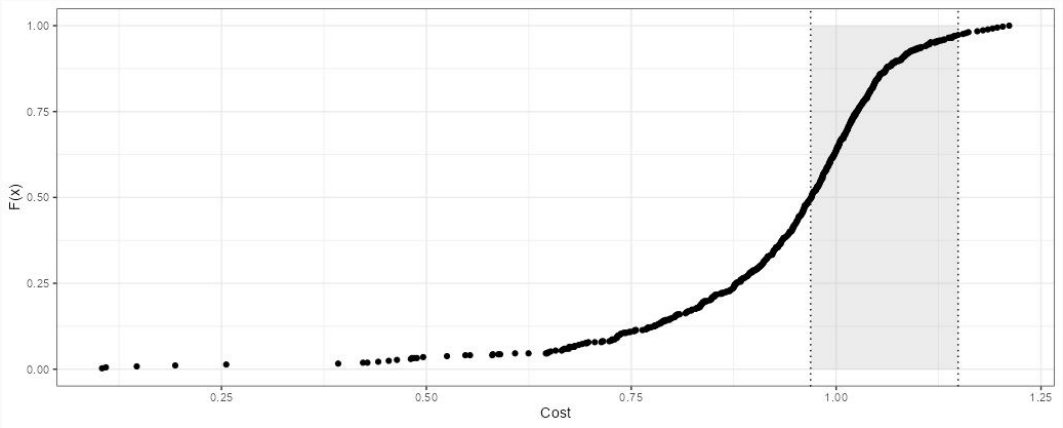
Descriptive Data

# Recovering the Cost Distribution and Convexifying it

- Cartel firms bid in the competitive markets as well.
  - DOJ and Caoui (2022) show that in these markets there is no proof of collusion.

- We apply a two-step process on the bids from the competitive markets: ▸ 1. ▸ 2.
  1. Concentrate-out the observed cost components from the bids to get residualized bids, applying Hickman and Hubbard (2015).
  2. Estimate the CDF and PDF of the residualized bids and apply Guerre et al. (2000) to recover the cost distribution.
- We convexify the distribution by smoothing and stabilizing the distribution in the sparse parts. ▸ Convexification

# Empirical CDF of Residualized costs

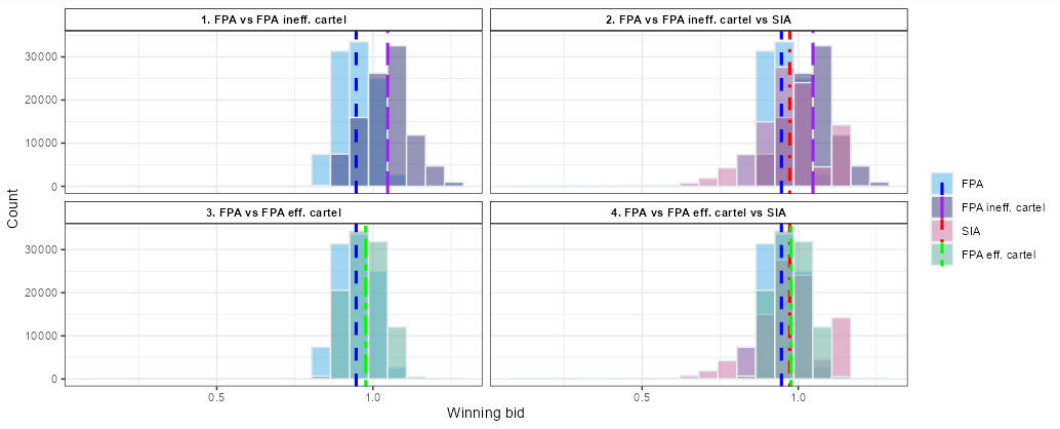



ECDF $tol = 0.0$

# Monte Carlo Analysis

- To assess the efficiency of SIA we perform a MC analysis in a $n = 3$ auction.
  - We make 3 random draws from the estimated cost distribution.
  - Collusive bids based on efficient and inefficient cartel bidding. ▸ Definitions
  - FPA competitive bids based on the optimal bidding function implied from bids' CDF.
  - Find the winning bid and price under each auction mechanism.
- We repeat this process 100,000 times.

# Monte Carlo Results I - Histograms



# Monte Carlo Results II - Averages

Average Winning Bid

| FPA | SIA | FPA ineff. cartel | FPA eff. cartel |
|---|---|---|---|
| 0.946 | 0.968 | 1.042 | 0.977 |

- SIA is on average 1 % cheaper than an efficient cartel.
- SIA is on average 7.6 % cheaper than an inefficient cartel.

# Applying SIA in the DFW Milk Market Cartel

- We compare the actual winning prices in the collusive auctions with the simulated winning prices under SIA. ▸ Actual-Predicted

- We construct simulated SIA bids as: ▸ 1.

$$\tilde{b}^{SIA} = exp(\mathbf{x}_d'\alpha)c_{id} \quad c_{id} \sim F_{\tilde{c}}$$

- $c_{id}$ is a random object. We make 10,000 draws.

# Cost Reduction under SIA

- We compute:

$$\text{cost savings}_d = \beta_d^{C*} - \text{mean}(\tilde{b}_d^{SIA*})$$

- We average across auctions $d$.

Average Cost Savings under SIA

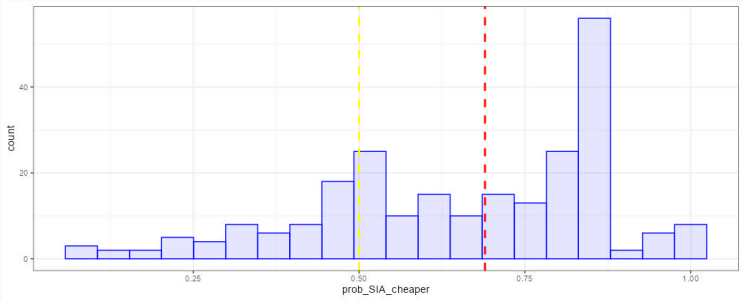
| Average Saving | Median Saving | Average Saving (pct) | Auctions with Reduction (pct) |
|---|---|---|---|
| 0.0065 | 0.0067 | 4.6156 | 80.0830 |

cost savings hist.

# Winning Bids Distribution

- We compare the actual winning price with the SIA winning price for each auction.
- We get the share of SIA winning prices that are lower than the actual winning price for each auction.

Histogram of Shares of SIA Winning Prices Lower than the Actual Winning Price



# Discussion

**Theory**

- We show that a Modified Vickrey Auction with randomization is strategy-proof and weak-collusion-proof,

**Empirical**

- We estimate the cost savings that this auction could generate in a collusive market.
- SIA decreases the price in around 80% of the actual collusive auctions.
- A back-of-the-envelope calculation gives a cost-savings per year and of 8.8% of the average total value of milk purchase (Average 1,804,514 USD dollars 1980).

Thank you very much!

# Appendix

# Assumption 1

Back

**Assumption 1 (A1).** Previous tender information.

- Auctioneer gets access to previous tenders, and aggregates the bids,
- From these previous bids, the auctioneer recovers the previous cost optimal strategy of the bidders,
- From the optimal strategies, the aucioneer recovers the distribution $F_0$ of marginal costs.

# Observed Contract heterogeneity



- $\mathbf{x}'_d$ contains: time dummies, number of bidders in auction $d$, price of raw milk #meals served, #meals served squared, #deliveries per week and #schools in the school district.
- all continuous variables are in $log$s.
- Time dummies capture common packaging, processing and labor costs that affect all the bidders.

# Control for Observed Contract heterogeneity

Back

- To control for contract heterogeneity we follow Haile et al. (2003).

$$\beta(c_{id}) = \beta(\tilde{c}_{id})\Gamma(x_d) \text{ with } \Gamma(x_d) = exp(\mathbf{x}_d'\alpha)$$

- OLS regression based on the the competitive areas' auctions.
- The residuals of this regression give us the $\tilde{\beta} = \beta(\tilde{c}_{id})$.
- We add back the "effect" of the $\#bidders$ to create $\#bidders$-specific distr.
- We concentrate our analysis on auctions with 3 and 4 bidders.

$x_d$

# Recover the Cost Distribution

Back

- Recover the distribution of the "residualized" cost from the inverted FOC.
  - Applied separately for $n = 3$ and $n = 4$ auctions.

$$\tilde{c}_{id} = \tilde{\beta} - \frac{1}{N-1} \frac{1 - \hat{F}_B(\tilde{\beta})}{\tilde{f}_B(\tilde{\beta}; \tilde{\beta}^{min}, \tilde{\beta}^{max})}$$

- $\tilde{f}_B(.)$ is the boundary corrected PDF of the "normalized" bids following Hickman and Hubbard (2015).
- We estimate non-parametrically the CDF of $\tilde{c}_{id}$
- We test the statistical equality for $n = 3$ and $n = 4$. KS test
  - We pool the retrieved cost distributions.

# Computation of the Concave Intervals

Back

- The SIA mechanism described above requires the concave parts of the costs' CDF.

1. Estimate a smoothing spline over the grid points of the costs' PDF $f^s_{\tilde{c}}$.
2. Take the derivative of the smoothed PDF $f^{s\prime}_{\tilde{c}}$.
3. Stabilize near zero derivative values. $f^{s\prime}_{\tilde{c}} = \begin{cases} 0 & \text{if } |f^{s\prime}_{\tilde{c}}| < tol \\ f^{s\prime}_{\tilde{c}} \end{cases}$
4. Identify the concave parts when $f^{s\prime}_{\tilde{c}} < 0$.
5. If non-concave intervals appear between two concave ones and their width is smaller than a chosen threshold merge them .
6. Find the exact points where $f^{s\prime}_{\tilde{c}} = 0$ within a smoothed bracketing interval.

# Concavification of the Probability of the Minimum

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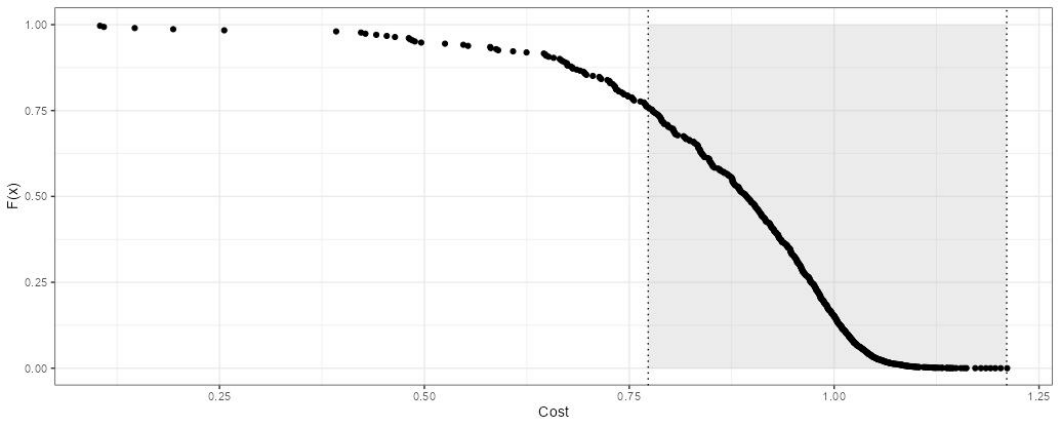

# Statistical Equality of Recovered Cost Distributions

Back

- We test the statistical similarity of the recovered CDFs of $\tilde{c}_{id}$ for $n = 3$ and $n = 4$.
- We apply the Kolmogorov-Smirnov test.

KS Test

| statistic | p_value |
|---|---|
| 0.074 | 0.411 |

# Emprical CDF of Residualized costs at exact 0.0

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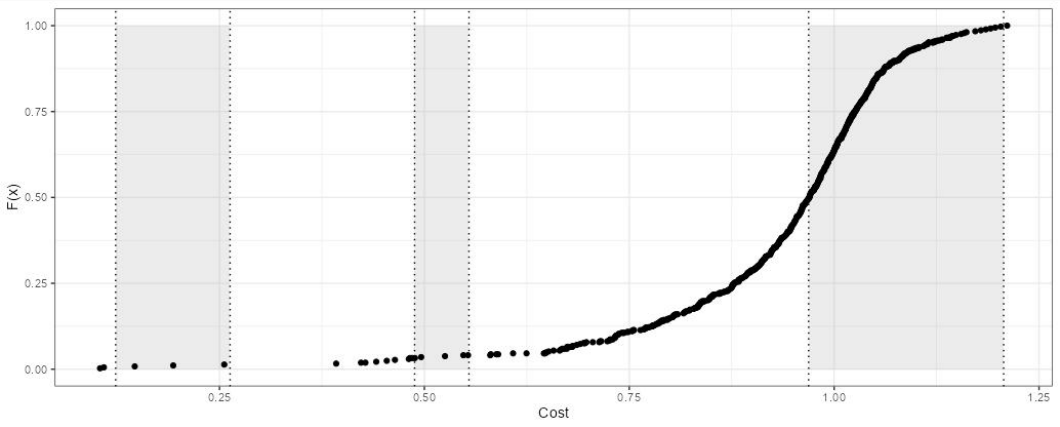


# Efficient and Inefficient Cartels



**Definitions**

- **Efficient cartel:** the cartel member with the lowest marginal cost is the one with the contract awarded.
- **Inefficient cartel mechanism:** A cartel where the bidder getting the contract is not the one with the lowest cost.

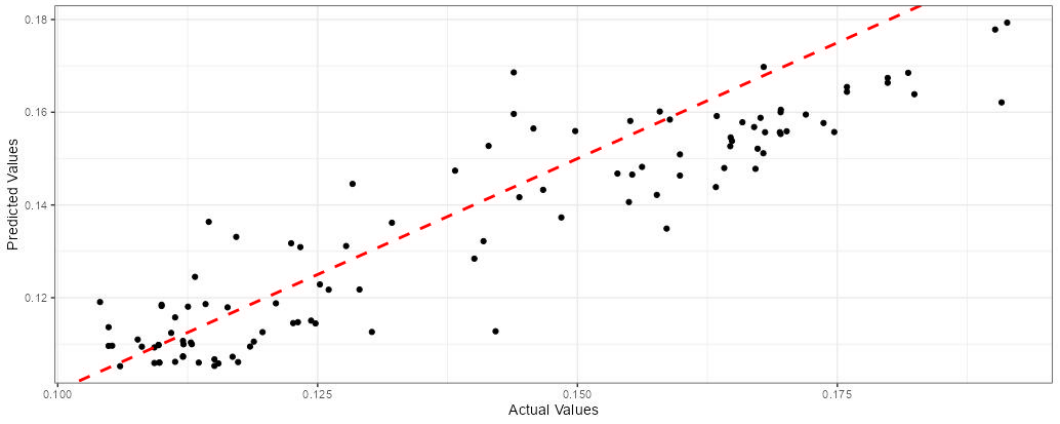
**Implementation**

- **Efficient cartel:** winning bid is equal to the marginal cost (of the most efficient firm) plus an overcharge.
- **Inefficient cartel:** random selection of the winner, the winning bid is his own marginal cost plus an overcharge.

Cartel overcharge is estimated through the data and it is 3.23%.

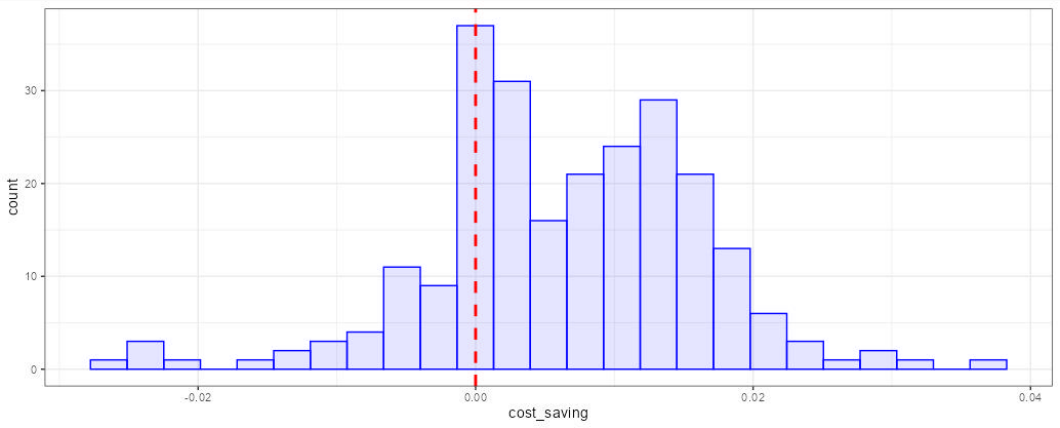
# Actual *vs.* Predicted First Price Auction

Back



# Histogram of Cost Savings under SIA

Back



# Descriptive Statistics

Bids' Descriptive Statistics

| #bidders | #obs comp | $\beta_{comp}$ | sd($\beta_{comp}$) | $\beta^*_{comp}$ | sd($\beta^*_{comp}$) | #obs coll | $\beta_{coll}$ | sd($\beta_{coll}$) | $\beta^*_{coll}$ | sd($\beta^*_{coll}$) |
|---|---|---|---|---|---|---|---|---|---|---|
| 2 | 248 | 0.1424 | 0.0248 | 0.1387 | 0.0232 | 537 | 0.1445 | 0.0239 | 0.1415 | 0.0212 |
| 3 | 453 | 0.1438 | 0.0264 | 0.1395 | 0.0260 | 865 | 0.1508 | 0.0260 | 0.1456 | 0.0233 |
| 4 | 276 | 0.1419 | 0.0273 | 0.1351 | 0.0274 | 593 | 0.1526 | 0.0222 | 0.1450 | 0.0215 |
| 5 | 134 | 0.1646 | 0.0653 | 0.1476 | 0.0250 | 272 | 0.1503 | 0.0132 | 0.1436 | 0.0131 |
| 6 | 42 | 0.1638 | 0.0263 | 0.1458 | 0.0320 | 110 | 0.1627 | 0.0154 | 0.1534 | 0.0136 |
| 7 | | | | | | 27 | 0.1627 | 0.0083 | 0.1528 | 0.0096 |

- $\beta$ is a bid in an auction in a given market.
- $\beta^*$ is the winning bid in an auction in a given market.

Back

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